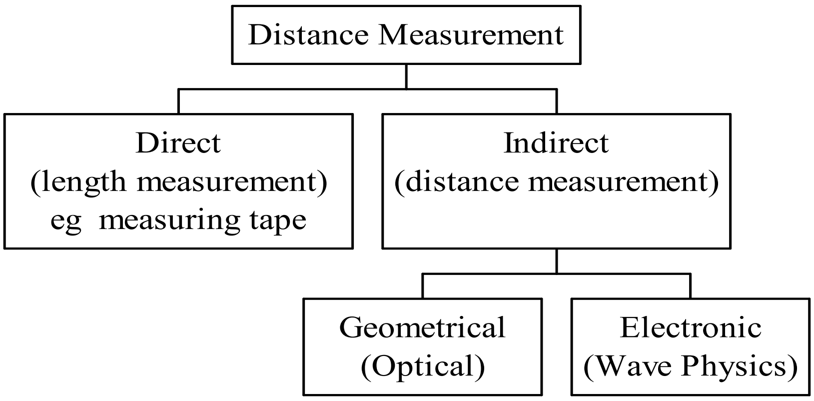
CHAPTER 2

2.0 DISTANCE MEASUREMENTS

One of the basic operations of surveying is the determination of natural and man-made features on a plane projection surface, i.e., determination of horizontal coordinates. This is usually accomplished by measuring i) horizontal distance from a known point and ii) orientation from a known reference direction.

2.1 Type of Distance Measurements

One of basic components of planimetric surveys is the determination of the distance between two points on the surface of the earth. In surveys of limited extent (remember principles of plane surveying) the distance between two points at different elevations is reduced to its equivalent horizontal distance either by the procedure used to make the measurement or by computing the horizontal distance from a measured slope distance (Moffitt et al., 1987).



As can be seen, these methods differ in type instrument, in the cost, in the length of the line as well as in the accuracy or quality. This chapter will emphasize the use of

- ✓ taping (direct method)
- ✓ tacheometric measurements (indirect/optical methods)
- ✓ EDM and total stations measurements (electronic methods)
- ✓ GPS surveys (space methods)

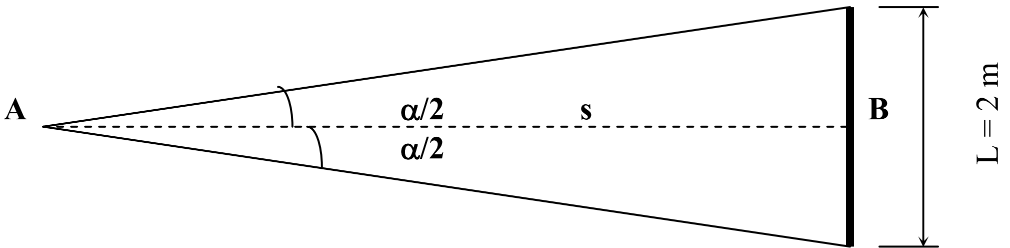


Figure 2.1 Substense bar

Of course, there are other techniques that may be used for determining distances between two points. They usually use trigonometric principles and derive distances indirectly. One example is substence bar, where the horizontal distance is determined by measuring horizontal angle to a fixed bar (usually 2 m), which is set on a tripod horizontally and perpendicular to the line to be measured. From fig. 2.1, the horizontal distances between A and B may be determined as,

$$s = L / 2 \cot \alpha / 2$$

2.1.1 Taping

One family of group of distance measuring instruments consists of tapes, chains, lines, rules, rods, and other calibrated lineer devices which can be laid end for end to accumulate a distance between points. The ancient surveyors used ropes, lines, cords, rods fixed in length and so on. They even used pacing when approximate results were found to be satisfactory. In the last case, to find the distance, the number of paces are counted and then multiplied by the pace length.

The tapes used in distance measurements are usually made of steel or invar, and rarely of cloth. They come in different lengths such as 20m, 30m, 50m and 100m. The short ones are preferred for ease of operation. For the distance measurements by taping, there are two techniques used in practice: horizontal taping and slope taping. Common to both techniques, there are some accessories used such as range poles (with their stands), plumb bobs, pins, etc. The horizontal distance between two points can be obtained with a tape either by keeping the tape horizontal or by measuring along the sloping ground and computing the horizontal distance. The latter method is advantageous in applications where extreme precision is required or steep slopes are encountered.

2.1.1.1 Horizontal Taping: It is the distance measurement between two points with a tape by keeping the tape horizontal. The procedures of horizontal taping may be summarized as follows. First of all, 2 range poles are set at both ends of the line to be measured (A and B in Figure 2.1). If needed, additional poles are set along the line. The forward tapeman and the rear tapeman measure the distance in the following fashion (consider Figure 2.2).

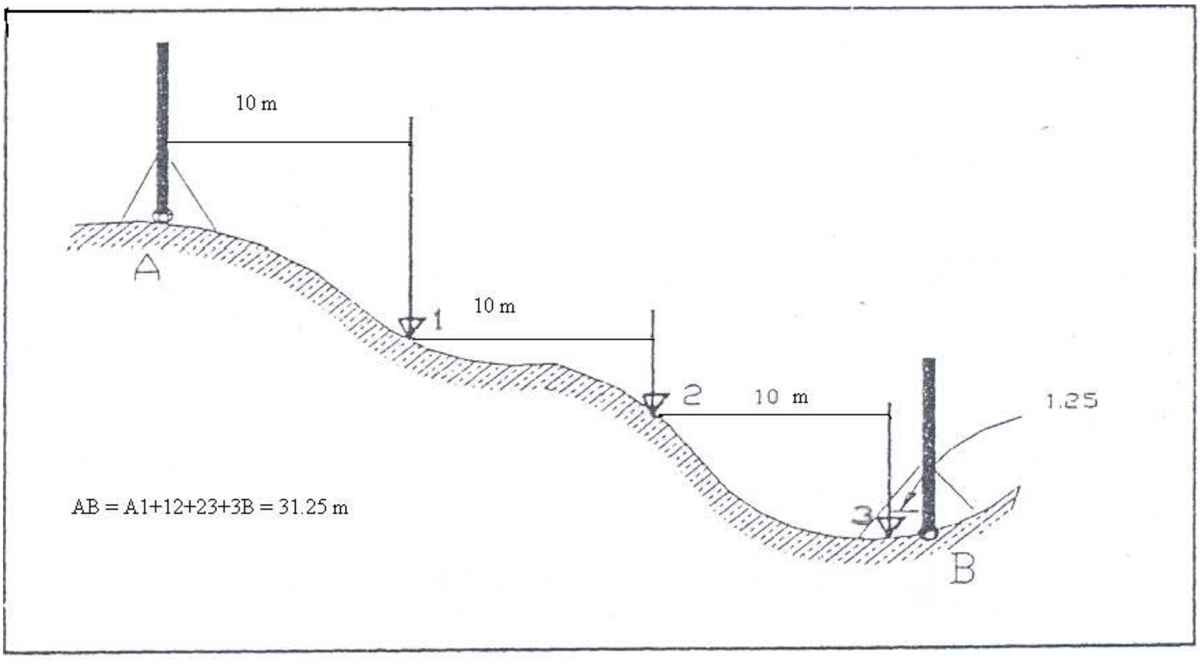


Figure 2.2 Horizontal Taping

a) The rear tapeman holds 0-end of the tape at A and brings the forward tapeman into the AB direction (alignment)
b) The forward tapeman holds the tape horizontal and projects the tape reading to the ground by means of the plumb bobs and marks the point 1 by taping pins.
c) Similarly the lengths 12 and 23 are measured.
d) The 3B length is measured by holding the tape between knee height and waist height by means of the plumb bobs.

The total horizontal distance between A and B is the sum of tape measurements, i.e.

AB = A1+12+23+3B
= 10+10+10+1.25 m
= 31.25 m

For moderate precision, where the ground is level and fairly smooth, the tape can be stretched directly on the ground and measurements may be compiled.

It is of utmost importance in horizontal taping that i) tape is horizontal ii) tape is stretched and straight iii) all the points are along the same line (A, 1, 2, 3 and B in Fig. 2.2).

2.1.1.2 Slope Taping: Where slopes are considerable and the ground surface is smooth but inclined, then taped measurements are directly made as stated in previous part, but with the tape on the ground or parallel to the ground. The slope distances are then reduced to the corresponding horizontal distances using the slope angle measured by a theodolite or determined by other means (see Fig.2.3).

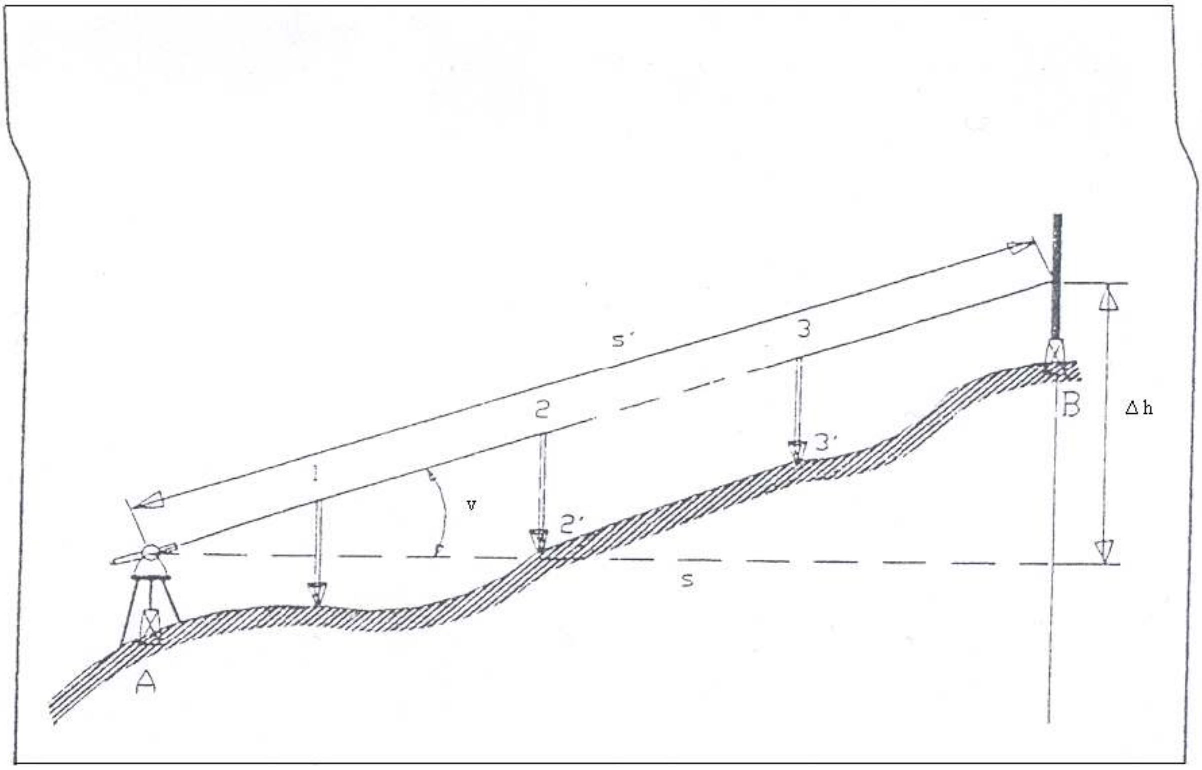


Figure 2.3 Slope Taping

If the measured slope distance and slope angles are denoted by s′ and v respectively, then the horizontal distance s is given by,

$$s = s' \cos v \qquad (2.2)$$

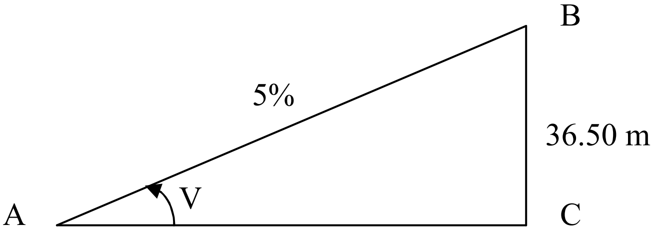
Here, s′ is measured in 4 pieces, i.e.

$$s' = A1+12+2'3'+3B$$

The vertical angle v is also known as slope angle and given percentagewise. It may be defined as,

$$\tan v = \Delta h / s \qquad (2.3)$$

Example: If the elevation difference between A and B is +36.50 m and the slope is 5%, what is the horizontal distance?



From Eq. (2.3),

$$\tan v = 0.05 = 36.50 / AC$$

Hence, AC = 730.00 m

It is important that the slope angle is given accurately. To determine the precision with which the vertical angle must be measured in order to meet a given relative accuracy in the horizontal distance, differentiate the above expression with respect to v, thus

$$ds = -s' \sin v \, dv \qquad (2.4a)$$

and the standard error is,

$$\sigma_s = s' \sin v \, \sigma_v \qquad (2.4b)$$

Example: A measurement of 323.69 m is made along a line that is inclined by a vertical angle of 3°22′ ± 3′37″ as measured using a theodolite. What is the corresponding horizontal distance and its error.

Using Eq. (2.2) and (2.4b) we have,

$$s = 323.69 \cos 3°22' = 323.13 \text{ m}$$
$$\sigma_s = 323.69 \sin 3°22' \;\; 217''/\rho'' = 0.02 \text{ m}$$

2.1.1.3 Errors in Taping: Errors in linear measurements made with tapes are:

- Gross errors
  - personal
- Systematic errors
  - standardisation - calibration over time
  - tension - manufactured and calibrated at a set tension
  - temperature - manufactured and calibrated at a set temperature
  - sag - in catenary the tape will sag under its own weight
- Random errors
  - Plumbing, marking, interpolation

Standardizations of taping;

- tape has a nominal length under certain conditions
- over time a tape stretches
- standardisation needs to be carried out frequently
- use reference tape or baseline
- L = recorded length of line
- l = nominal length of field tape (eg 30m)
- l' = standardised length of field tape (say 30.011m)
- sign of correction depends on the values of l and l'

Formulation of the above errors and corrections to tape measurements accordingly can be found in conventional surveying text books (e.g. Moffitt et al., pp. 33-40). As precise distance measurements are made by EDM's, total stations, GPS techniques nowadays, the authors do not see any application where a civil engineer will use taping and require to have the corrections mentioned above. Therefore, they will not be covered here anymore. The students, however, must do their best to minimize errors due to incorrect alignment, tape not straight, and tape not horizontal. It is left to students to derive the magnitude of the errors due to these problems. For example: in a 100 m measurement, an alignment error of 1.41 m (off the line) would produce an error of 0.01 m in the horizontal distance; if the middle point of a 50 m tape is 1.42 m off the line, the resulting error in length is 0.01 m.

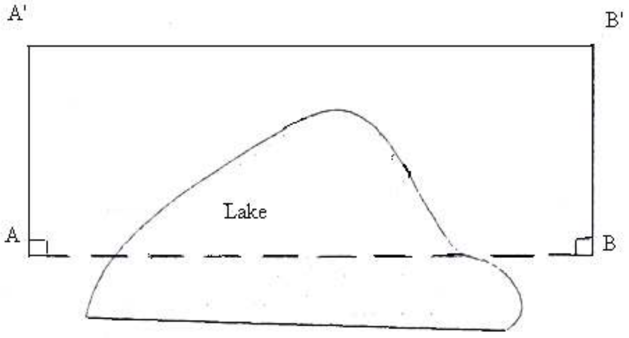
2.1.1.4 Measurement of Obstructed Distances:

So far, the direct measurements of distances have been covered. There are cases where direct measurements may be very difficult or impossible due to obstructions. In such cases, one can apply conventional indirect measurement techniques such as parallel offset, swing offset, long offset, similar triangles etc. They are nothing else but methods utilizing simple geometry an trigonometry.

Parallel offset

Set A′ and B′ by erecting perpendicular to AB from A and B at an equal distance and then measure A′B′. Thus,

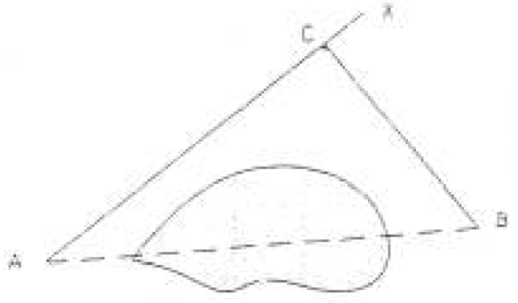
$$AB = A'B'$$



Swing offset

Set a point X and, then, erect perpendicular to AX from point B in order to find point C so that BC ⊥ CA. Now measure AC and CB to have,

$$AB = (AC^2 + CB^2)^{1/2}$$



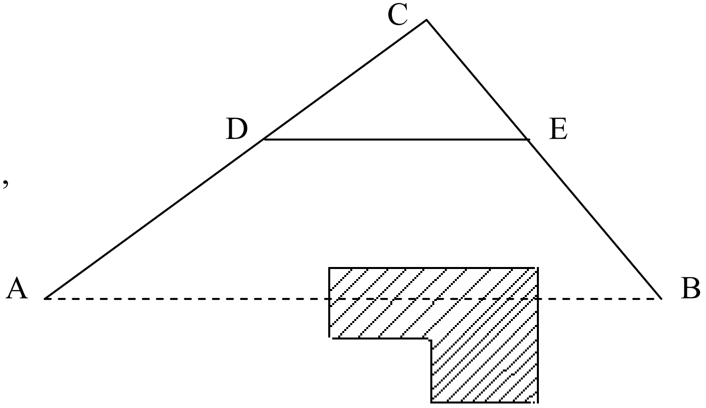
Similar Triangles

Fix a certain point C from where A and B are visible. Now, first measure CA and CB, and then mark D along CA and E along CB so that

AC/DC = CB/CE=c

Finally, measure DE and compute AB, i.e.,

AB= c DE



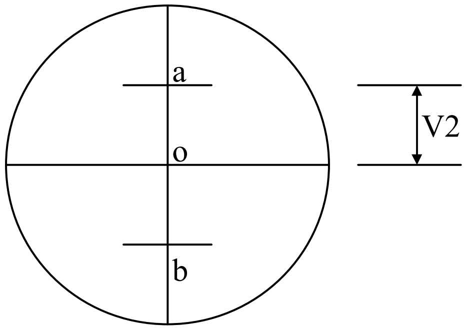
2.1.2 Optical (Tacheometry / Stadia) Measurements

The term tacheometry or stadia in surveying is used to denote the procedures for obtaining horizontal distances and elevation differences by indirect methods based on the optical geometry of the instruments employed (Moffitt et. al., 1987). The instruments commonly used in Turkey are theodolites and stadia rods. The precision of tacheometry is inferior to those of taping, EDM and GPS measurements. Stadia surveys are generally used to measure the lengths of side shots in planimetric measurements and to determine differences of elevation between points for topgraphic mapping and so on.

The principles of tacheometry by use of the theodolite and stadia rod will be presented here. One may simply define stadia rods as vertically held scales. They usually come in 2.0 m, 3.0 m and 4.0 m and are made in sections of 1 m. the graduations of stadia rods are generally in centimeters or millimeters. Distinctive marking designs are employed to facilitate reading. Theodolites are the most commonly used tacheometry devices developed in late 18th century. Basic components of a theodolite is as follows:

- ✓ Telescope
    - Objective lens
    - Eyepiece
    - Reticle with stadia (cross) hairs
- ✓ Horizontal and vertical circles
- ✓ Axes
- ✓ Levels

The vertical separation of the upper and lower stadia hairs in the reticle is designated as i. In the reticle to the right and Fig.2.4, points a and b represents the positions of the upper and lower stadia hairs with spacing ab=i. The middle hair (point o) is read to check the upper and lower readings as it is the mean of the said readings. Referring to Fig. 2.4, F is the principal focus of the objective lens with the focal length f.



Horizontal Stadia Measurements

Rays of light directed along the lines aa' and bb' (both parallel with the optical axis of the telescope) will continue in the direction a'FL and b'FU respectively. The vertical distance h=UL, which can be measured on a stadia rod by subtracting the stadia hair readings (i.e., U-L), is called the stadia interval. By similar triangles in Fig. 2.4, the distance r may be found from the equation, - see chapter 7 on stadia measurements.

2.1.3 Electronic Distance Measuring (EDM) and Total Stations Measurements

The last 40 years has been a revolution in the methods of measuring distances with the introduction of EDM instruments and satellites. The first EDM instrument was developed by Dr. Bergstrand in Sweden in 1946. It was called geodimeter and it employed a modulated light beam for determining distance. It was followed by an instrument called tellurometer developed by T. Wadley of South Africa in 1956. The tellurometer employed modulated microwaves for determining distance.

The early models were expensive, heavy and bulky, their use and maintenance generally required some skill beyond that of the ordinary surveying technician. Therefore, they were not used much until the 1970's. However, with modern circuitry and other developments, the equipment has gradually become less bulky and expensive and much easier to operate and maintain. The third generation of EDM's employing highly coherent laser light has been brought to perfection in recent years (Moffitt et. al., 1992). This type of instrument has the distinct advantage of long-range, low-power requirements, and fairly good portability as well as ease of operation and readout. There is hardly a progressive surveying firm today that does not employ EDM equipment to some extent, especially those using modulated light beams. Modern instruments are fully automatic to such extent that they may even have the capability of measuring horizontal and vertical angles as well as the slope distance, and displaying angles, both horizontal and slope distances, and elevation differences between the ends of the line. These instruments are known as total stations. The displayed outputs can be recorded on a REC Module or stored in electronic field books for further calculations in computers. Thus, manual data recording and data entry are avoided.

The instruments using modulated light beam employ frequencies in the visible light wavelength spectrum and the infrared range. The equipment consists of i) and electrical unit and ii) a retro prism. The electrical unit transmits the light to the prism which reflects it back to the unit for analysis of the double slope distance between the two points (Fig. 2.6). The returned light is converted to electrical current by a photo tube. The wavelength of the light is known and by a method of phase comparison, the partial wavelength can be determined.

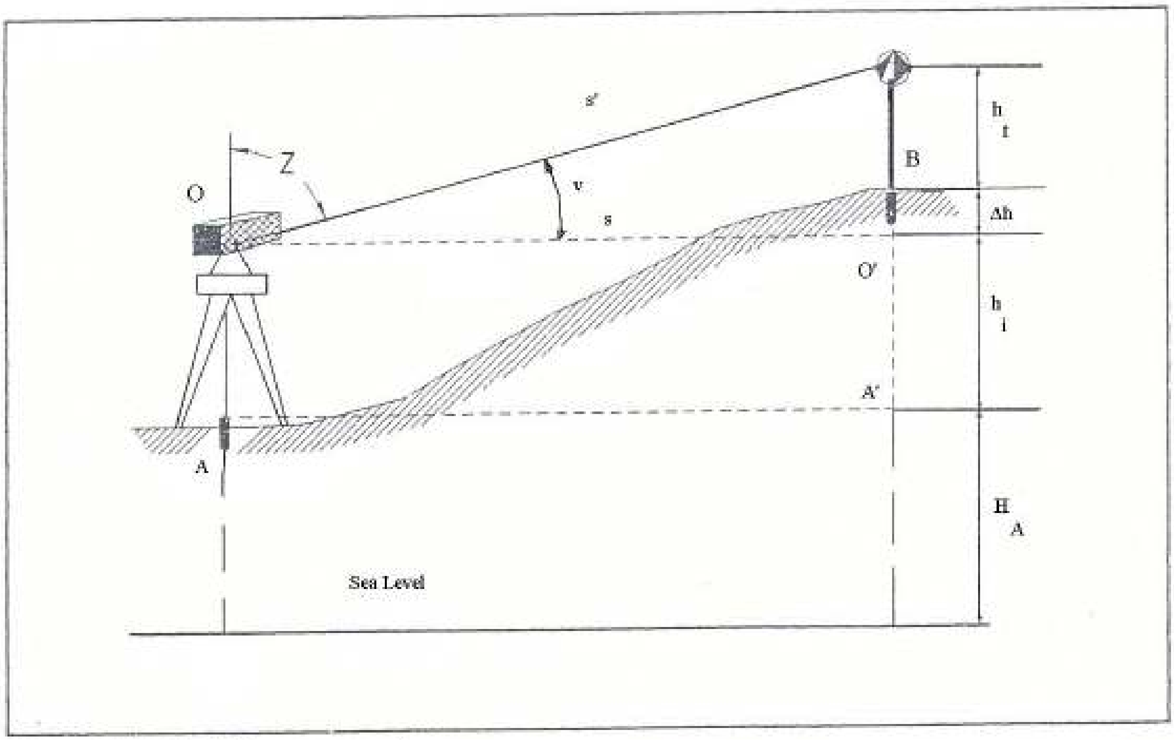
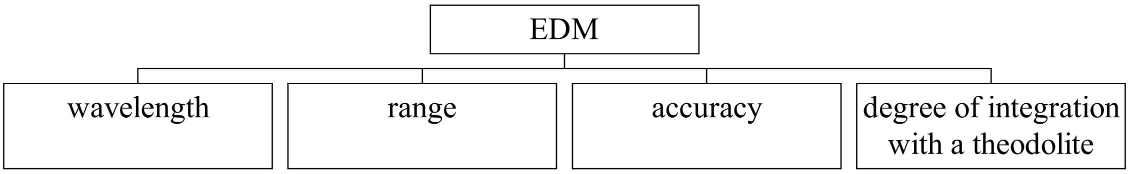
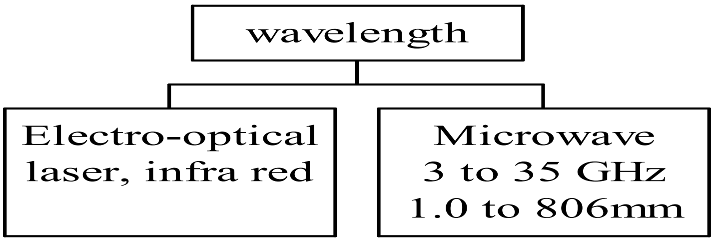
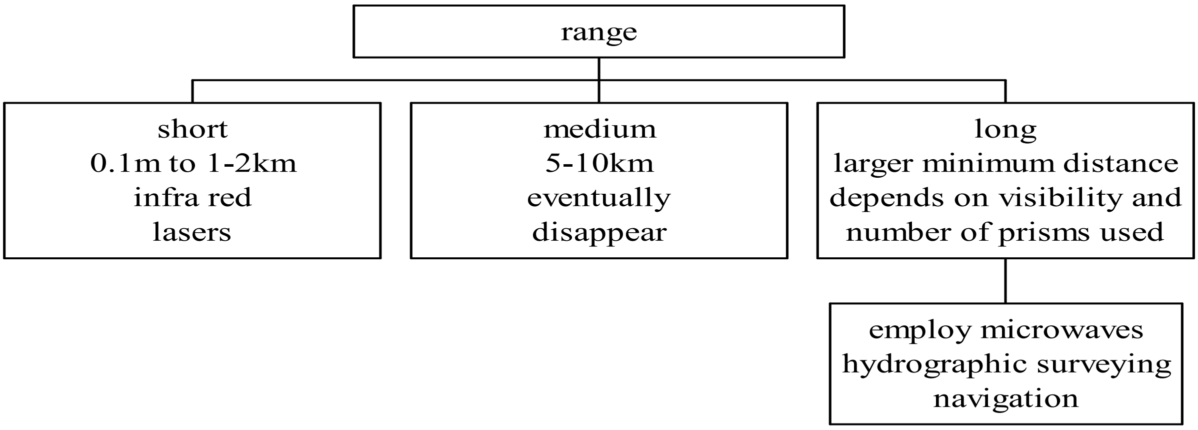


Figure 2.6 EDM Measurements

2.1.3.1 Classisfication of EDM's







The classification of instruments are usually made by their range capabilities:

a) <u>Short-range EDM</u>: Measures up to 3 kms depending on the numbers of reflectors used. The light carrier for all these instruments is infrared light in the 0.90 – 0.93 μm.
b) <u>Medium-range EDM</u>: Measures up to 10 kms depending on the numbers of reflectors used.
c) <u>Long-range EDM</u>: Measures up to 60 kms depending on the numbers of reflectors used. These instruments use a laser light as the carrier.

A tabulation of frequencies used for EDM – visible light to microwave – is provided below.

λ (meters)	***f (cycles)***	***Nature***	***Property***
3000-1000	100-300 KHz	Very long radio waves	Travels along the curvature
1000-100	300-3000 KHz	Medium	
100 -1	3 -300 MHz		
1-0.01	300-30000 MHz	Microwave	Wave starts travelling in straight line and penetration through denser medium incr.
$0.01-10^{-6}$		Infrared	
0.4 μm - 0.8 μm		Visible light	
10^{-10} - 10^{-12}		X-rays	Will penetrate

Now, let us see how EDM instruments measure distances. The slope distance between the EDM instrument and the prism (see Fig. 2.6) can be defined as,

$$s' = 1/2\,(n\lambda + \Delta\lambda) \qquad (2.11)$$

where, λ = the wavelength of modulation
n = the number of full wavelengths
$\Delta\lambda$ = the partial wavelength

Moreover, the wavelength is defined by,

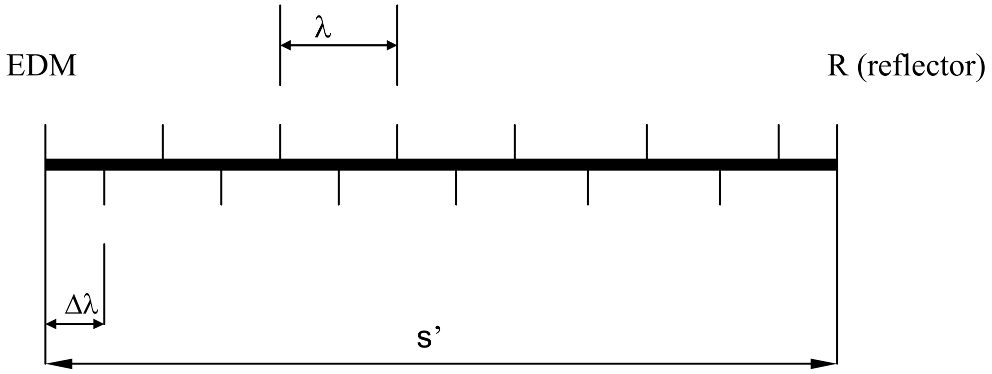
$$\lambda = c/f \qquad (2.12)$$

where, c = the velocity of light through the atmosphere affected by air temperature, pressure, and water vapor, etc.

f = the modulation frequency

2.1.3.2 The measurement principle of EDM

The measurement principle of EDM is illustrated in the diagram below.



The number n is solved by using more than one frequency and solving the equations simultaneously. Actually, this is all done automatically in modern instruments. One such system in common use is the decade-modulation technique. Suppose that we want to measure a distance of 485.276 m. Now assume that a modulation frequency of 15 MHz set up in EDM, resulting in a half wavelength of 10 m. Thus, the phase-meter reading gives *5.276 m* part. Switching down to 1.5 MHz, the half wavelength is now 100 m, which is resolved by the phase meter to give the tens of meters – in this instance 80m. The next frequency then is 0.15 MHz, which gives hundreds of meters, which in this instance is 400 m. The total of these readings would be 485.276 m.

2.1.3.3 Reduction of Slope Measurements in EDM

By EDM instruments, the slope distance s' between the EDM and the reflector is measured (Fig. 2.6). The corresponding horizontal distance may be computed by,

$$s = s' \cos v \qquad (2.13)$$

Similarly, the elevation difference ΔH_{AB} between A and B is determined from,

$$\Delta H_{AB} = s' \sin v - h_t + h_i \quad \text{and} \qquad (2.14)$$
$$H_B = H_A + \Delta H_{AB} \qquad (2.15)$$

If EDM and theodolite have different height of instrument and targets, required corrections must be made accordingly. In Figure (2.7), the EDM center is at E measuring to the reflector at R, the theodolite center at O measuring to the target point T. The theodolite measures the vertical angle v'. The correction Δv' must be added to v' to obtain the correct vertical angle v of the measured slope distance as used in the above expressions. Thus, we have,

$$\Delta v' = [(h_R + h_T) - (h_E + h_O)] \cos v'/s' \qquad (2.16a)$$

and,

$$v = v' + \Delta v' \qquad (2.16b)$$

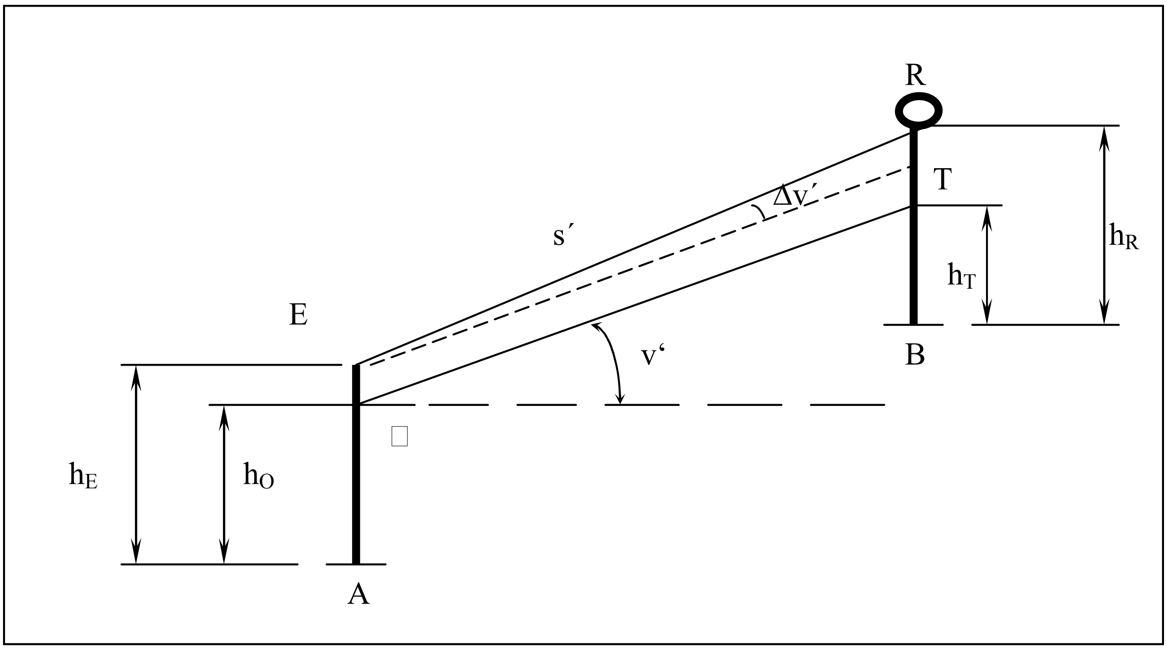


Figure 2.7 Difference between EDM and vertical angle directions

In the EDM distance measurements, one should watch out for the reflectors used; the constant of the reflector, c_R may not be equal to 0 (zero). In other word the reflective plane of the reflector may be off the center of the reflector, causing measurements too long or too short. Although modern prisms have the reflector constant as zero, there are still some reflectors in the market with non-zero reflector constant. Manufacturers provide the magnitude of these constants. They may be directly set in EDM instruments to have correct distance reading or they map be applied later in the office. It is also possible to determine prism constant by measuring a well known distance. Thus,

$$c_R = \text{Given distance} - \text{EDM measured distance} \qquad (2.17)$$

2.1.3.4 Effects of Atmospheric Conditions on Wave Velocity

The air temperature, the atmospheric pressure and the relative humidity affect on the velocity of propagation of light and microwaves. A knowledge of these conditions allows the determination of the refractive index of the air. For light waves the refractive index n_g of standard air is given by,

$$n_g = 1 + (287.604 + 4.8884 / \lambda^2 + 0.068 / \lambda^4)\, 10^{-6} \qquad (2.18)$$

in which λ is the wavelength of the light carrier in micrometers. For example; for red laser and infrared carriers λ values are 0.6328 μm and 0.900-0.930 μm respectively.

the refractive index n_a for light waves at conditions departing from the standard air can then be computed by,

$$n_a = 1 + [0.359474\ (n_g\text{-}1)\ p]\ /\ (273.2 + t) \qquad (2.19)$$

in which p= the atmospheric pressure(in mm of mercury) t = the air temperature (in degrees Celcius)

The velocity of the light wave in air, c_a is related to the velocity of light c_o in a vacuum by,

$$c = c_{o\,/}\, n_a \qquad (2.20)$$

in which the value of c_o is 299792.5 km/sec.

Thus, the modulated wavelength of a light can be computed from Eq. (2.15).

Problem: The modulating frequency of a red laser beam is 24 MHz. This beam travels in the air under the conditions of 26°C and atmospheric pressure of 759 torr. What is the modulated wavelength of the light?

The effects of temperature and pressure corrections are pretty significant, especially for long baselines, if these values differ from standard ones significantly. EDM Instruments / Total Stations usually come with manuals and charts for these corrections, which can be directly set in the instrument to read correct distances or they may be applied later in the office. Since atmospheric pressure may be defined in terms of elevations, pressure corrections are usually computed from elevation charts. For the magnitudes of the said corrections, let us give an example: a difference of 10°C in temperature and error of 25 mm of mercury in pressure both will introduce a relative error of 10 ppm each.

2.1.3.5 Peripherals of EDM

- Atmospheric pressure
  - pocket barometers
  - hand held barometers
- Atmospheric temperature
  - mercury in glass thermometers
  - platinum resistance thermometers
  - electronic thermistor thermometer
- Atmospheric Humidity
  - aspiration psychrometer
  - humidity sensor

2.1.3.6 Special Features of Modern Short range EDM

- on-board application of first velocity correction
- computaion of horizontal distance and height difference
- tracking mode
- audio signal
- automatic data recording
- computer assisted surveying
- setting out aids
- pointing aids

2.3.1.7 Calibration

Although modern EDM equipment is exceptionally well constructed, the effects of age and general wear and tear may alter its performance. It is essential therefore that all field equipment should be regularly calibrated. In the light of legislation on quality assurance, calibration to ensure accuracy of performance to the standards demanded is virtually mandatory. Importance of calibration;

- quality control
  - the significance of the corrections with respect to the work required of the instrument

  - whether the instrument is working within the manufacturer's specified □
  - whether the instrument requires a service
  - whether any systemmatic errors exist

- improvement of accuracy
  - applying corrections to measured values improves the accuracy

- legal metrology

Calibration Concepts;

- baseline calibration (rigorous, mathematical)
- field calibration (practical)

The errors have been classified under three main headings;

1. Zero Error
2. Cyclic error
3. Scale error

Basically, the electronic distance determination is based on an equation $D=Ko+KDa$ like other distance determination. This means that the rough reading Da has to be multiplied with a scale constant K and an addition correction Ko in order to yield the desired distance D.

The zero correction is to be checked as well as the scale constant. It takes into account the difference between electrical and mechanical zero positions at the instrument as well as the separation of the reflection point and the physical centre of the reflector and the propagation time reduction inside the glass body. While the latter reflector components remain constant, unless the reflector is changed, the instrumental components may change. This is caused mainly by phase in homogeneities of the transmitters and of the photo detectors The mutual influence of the signals of transmitting and receiving parts has to be mentioned in this connection as well. The latter leads to cyclic changes in Ko just like possible errors of the phase meter. In most cases, however, these changes can be kept sufficiently small. It is therefore advaisable to regularly check the zero correction Ko. As a rule, a very simple arrangement is sufficient: If the scale constant K equals 1, the following equations are valid:

$D=Ko+Da$; $D=D1+D2=Ko+Da1+Ko+Da2$

Which leads to

$Ko=Da-(Da1+Da2)$

An independent determination of the zero correction together with its accuracy requires a large number of calibration measurements. These are carried out most efficiently on a subdivided testline. Such a Field Calibration should fulfil the following requirements;

- An EDM instrument can be tested and calibrated for all errors one at time, or simultaneously using a baseline
- an easy method is to determine each error by a series of tests which should be performed in the following order
  - Test to determine the degree of cyclic error
  - Determine the value of the additive constant
  - Measure known distances to find the scale error

2.3.1.8 Prism

The reflector or prism is a corner cube of glass in which the sides are perpendicular to a very close tolerance. It has the characteristic that indecent light is reflected parallel to itself, thus returning the beam to the source. This is called a retrodirective prism or retro reflector.

The reflectors have a so-called “effective center”. The location of the center is not geometrically obvious because light travels slower through glass than air. The effective center will be behind the prism itself and is generally not over the station occupied. Thus there is a reflector constant or prism constant to be subtracted from the measurement. Some manufacturers shift the center of the EDM forward the same amount as the prism offset to yield a zero constant. All Wild/Lecia EDM’s are shifted forward by 35 mm.

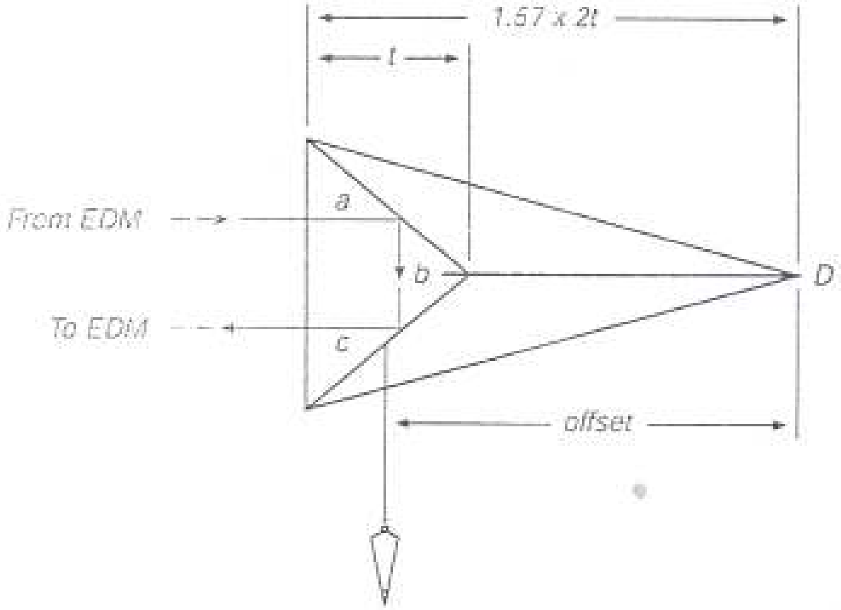


Figure 14

Reflector constant;

- EDM beam travels in air between instrument, reflector and instrument
- velocity of light in glass slower than in air
- correction due to non-coincidence between the reflecting plane and the prism house plumbing point
- can be combined with the additive constant for the EDM instrument

Always use prisms designed for your EDM system.

2.1.4 Global Positioning Systems (GPS)

2.1.5. Errors in Distance Measurements

Actually, there are two types of errors affecting measured quantities. They are systematic errors and random errors. A systematic error is the one the magnitude and algebraic sign of which can theoretically be determined. Example: let a line AB have the exact length of 20m and a tape is found to measure this length as 19.94 m, then the the full tape length introduces a systematic error of +0.06 m each time it is used for 20 m interval. A random error is the one the magnitude and sign of which cannot be predicted. Example: in taping, operator tries his best to estimate last digits, but it varies from person to person or even for the same person.

Systematic errors may be eliminated prior to post-computations. Random errors, however, cannot be eliminated, but they can be minimized by adjustment such as least-squares computations. In Table 2.2, error sources and accuracies for the different types of distance measurements are tabulated.

Methodology	**Systematic Errors**	**Random Errors**	**Precision**
Taping	- Incorrect tape length - Tape not horizontal - Incorrect temperature calibrat. - Incorrect tension or pull - Sag in the tape - Incorrect alignment - Tape not straight	- Observer reading error - Failure to apply proper tension - Error in temperature meas. - Wind deflecting plumb bob - Incorrect setting of taping pin - Plumb bob not steady	1 cm + 250 ppm
Stadia Measurements	- Incorrect stadia length - Incorrect stadia interval - Rod not vertical aligned	- Observer reading error - Stadia rod not steady - Changes in atm. Conditions	1 dm + 5000 ppm
EDM and Total Station Measurements	- Incorrect reflector constant - Incorrect temperature - Incorrect pressure	- Uncertainty in atm. Conditions	5 mm + 3 ppm
GPS Surveys	- Incorrect scale factor - Ephemeris Errors	- Uncertainty in atm. Conditions - Satellite clock errors - Receiver clock errors - Synchronization errors - Ephemeris errors - Error in phase readings - Multipath errors	5 mm + 1 ppm

Table 2.2 Errors in distance measurements

Distance Measurement

1. A slope distance of 176.20 m is measured between two points with a slope angle of $3^o32'$. Compute the horizontal distance between the points.
2. In Problem 1 if the vertical angle is in error by 2’, what error is produced in the calculated horizontal distance?
3. The difference in elevation between two points is 26.264 m. The measured slope distance is 583.804 m. Compute the horizontal distance.
4. A line was measured along a sloping ground with a 30 m steel tape and the following results were recorded:

Section	Slope Distance (m)	Difference in elevation (m)
A1	30	1.22
12	30	0.83
23	27	1.06
34	30	2.16
4B	12	0.56

What is the horizontal length of the line?

5. A tape that measures 29.992 m between the zero and 30 m mark is used to lay out foundation walls for a 120.00 m x 250.00 m. what observed distances should be laid out?

6. A building line is to be established 60 m from and parallel to the centerline of a street. The surveyor measures 15.00 m horizontally from the centerline to the edge of the street. Then he lays off the remainder of the distance along a $2^{o}30'$ slope. Calculate this remaining slope distance, to the nearest mm.
7. What distance should be laid out with a tape that measures 20.012 m under the prevailing field conditions if a 500 m is to be established?
8. What distance on a 6% grade should be laid out with a tape that measures 30.009 m under field conditions if the horizontal distance is to be 750.00 m?
9. The distance AB is measured along a 1 in 15 slope and found to be 212.165 m. Compute the horizontal distance.