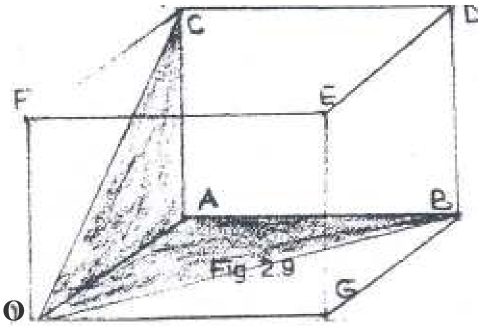
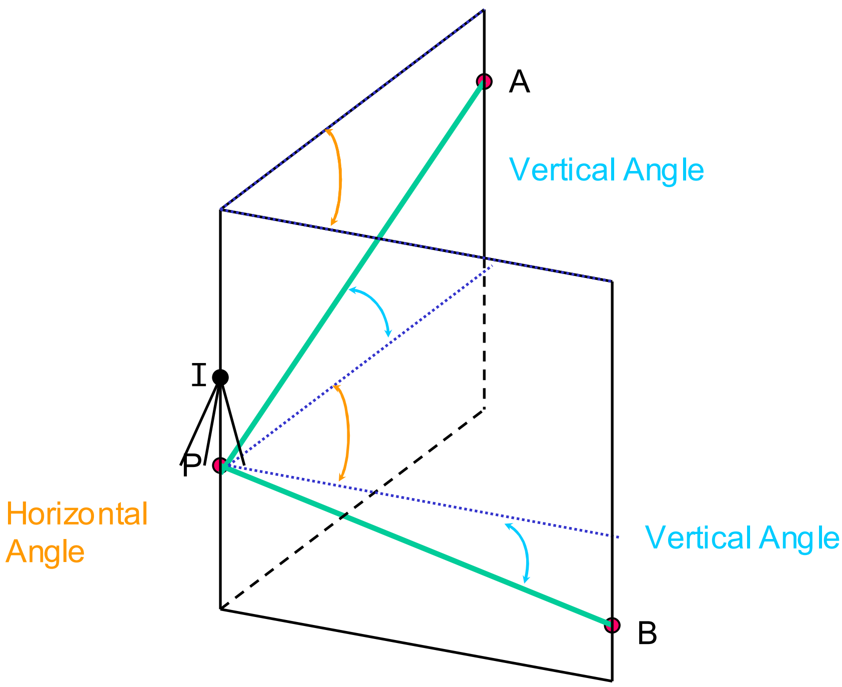
3. MEASUREMENT OF ANGLES

The second quantity which is necessary for determination of location of natural and artificial features as well as to lay out points for establishing civil engineering structures as specified in projects is horizontal angel between lines or direction of lines from a reference line. A horizontal angle is the angle formed by two intersecting planes. These vertical planes intersect along a vertical line which contains the vertex of the angle.

Five kinds of measurements illustrated in Fig 2.9 form the basis of plane surveying: (1) horizontal angles, (2) horizontal distances (OA,OB), (3) vertical angles (AOC), (4) vertical distances (AC,BD) and (5) slope distances. Horizontal angles as AOB are measured in horizontal plane.



The horizontal angle AOB is the angle between vertical planes OFCA and OFDB. The horizontal angle defined by OC and OB is equal to the angle AOB and is shown as horizontal angle COB. The inclined angle COB is called position angle and neither measured nor used in surveying.



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3.1.1 Position of a Point

Two points say A and B can be chosen in the field and the distance between them can be measured. The line AB and the points A and B now can be used as reference to determine the location (relative position) of a point P (or to lay out points as defined in the project). As shown in figure 2.10.

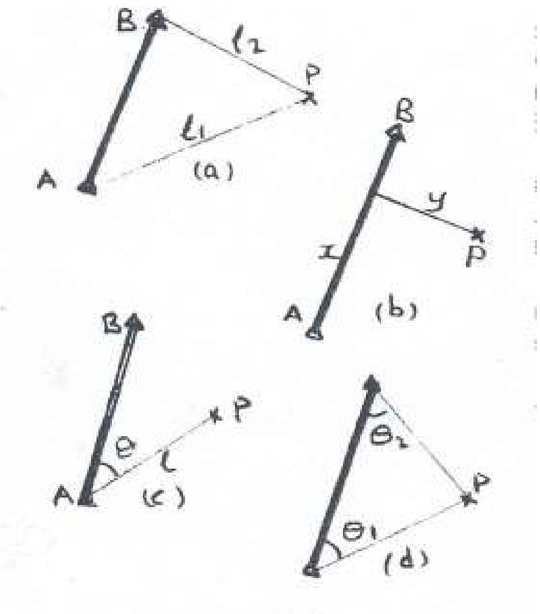


Fig. 2.10

Measure AP and BP from A and B respectively to locate P. Intersection of arcs with radii AP and BP with centers at A and B will locate P. Method is called trilaleration and is finding large application with modern EDM and GPS methods.

Measure PD at right angle to AB and measure either AD or BD. This method is called offsetting and is used in combination with other methods to locate subsidiary points required for defining details.

Measure AP and horizontal angle θ=BAP. This method is used in traverse surveys as well as detail surveys carried by stadia surveying.

Measure angles θ_1 and θ_2. This method forms basis of triangulation surveys where linear measurements (distance) are kept at minimum. With modern EDM and GPS techniques distances can be measured at required high accuracies economically and accordingly triangulation is leaving his place to trilaleration.

In the same survey several of the above methods may be employed using different combinations of instruments. It is important to note here that the work done in the field should be self-checking for mistakes and blunders. This can be achieved by doing extra measurements or repeating the work: like measuring θ_3 in (d).

3.1.2 Angle By Measured Distances

Angles are measured directly in the field by compass, theodolite or transits. As seen before some simple angle measurement or lay out can be carried out by using tape and accessories. Another method of doing this is to divide the survey area into a series of connected triangles, the sides of which are measured. From the lengths. of resulting triangle sides the angles at vertices are computed by using the formulas of plane trigonometry.

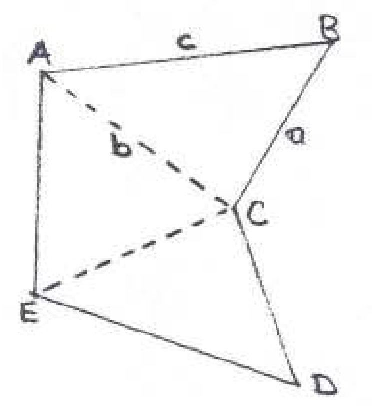


Fig. 2.11

For example, in triangle ABC

$$\sin(\alpha/2) = \sqrt{\frac{(s-b).(s-c)}{b.c}} \qquad (2.37)$$

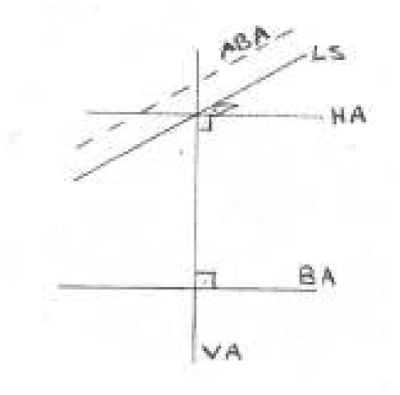
where a. b and c are sides of triangle ABC and

$$s=\frac{1}{2}(a+b+c)$$

Various separate angles as computed above are added to obtain the whole angle at each station (vertex). Single angles can be computed or laid out by using isosceles or right triangles as done in the second field work.

3.1.3 Theodolites

The theodolites are perhaps the most universal surveying instruments. Although their primary use is for accurate measurement or lay out of horizontal and vertical angles, they are also commonly employed for a wide variety other tasks such as determining horizontal and vertical distance by stadia, prolonging straight lines and low order differential leveling, ranging across obstacles, etc.



A theodolite is, simply a telescope which can be rotated in vertical and horizontal planes and these rotations can be measured in respective graduated circles as vertical and horizontal angles. To obtain these rotations the theodolite is equipped with bubble axis (BA), vertical axis (VA), horizontal axis (HA), line of sight (LS) (also called optical axis) and attached bubble axis (ABA), as shown in fig. 2.12. Bubble axis BA is provided by a tube bubble and is used to level (to make horizontal) the instrument so that vertical axis will be really vertical. VA is a mechanical axis around which telescope (LS) rotates in horizontal plane. Horizontal axis is also a mechanical axis around which telescope rotates in vertical plane.

The following relations among axes of theodolite must be maintained in order to assure accurate measurements:

VA□BA
HA□ VA
LS HA
VA.HA LS meet at O (center of instrument)
LS//ABA

LS is defined by cross hair in the reticle of telescope. The relations (2.38) can only be satisfied with certain specified tolerances (allowable errors). Magnitude of these tolerances will effect cost. Weight accuracy as well as operation of the instrument by the operator.

Depending on their system on reading of fractions of angles, theodolites are classified into two main groups:

a) vernier theodolites (transits), and
b) optical theodolites

In vernier theodolites which are called transits the fraction of angles are read on mechanical vernier scales. Transits are old type of instruments with large diameter horizontal circles and are used mostly in USA.

Optical theodolites give reading of fraction of angles (min.sec, c, cc, etc.) on optical micrometer and accordingly optical theodolites or simply theodolites are light weight instruments.

Another classification of theodolites is based on the number of horizontal motions (around VA) provided by the instrument.

a) Repeating theodolites have two movable horizontal plates thus providing one angular motion and another general motion without changing the angle reading on horizontal circle. They are also called upper (angular) motion and lower (general) motion of the instrument.

b) Direction theodolites have only one movable plate and provide a single horizontal (angular) motion. They measure circle readings on different directions. The difference in direction readings between any two stations is the angle.

Though repeating theodolites are more versatile and easy to operate, direction theodolites are preferred for very accurate and precise angle measurement.

Theodolites can also be classified as;

a) mechanical-optical theodolites which are the classical instruments of angle measurement mentioned above and
b) electronic theodolites give circle reading automatically on a digital electronic scale. The mechanical operation of electronic digital theodolites is similar to that for standard instruments.

Parts of theodolites and other details for operating the instruments will be covered during the field works on angle measurement.

3.1.4 Measurement of Angles by Theodolites

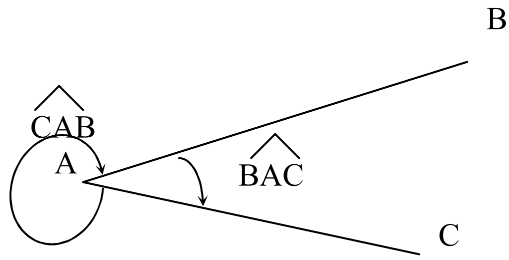
When they are centered over the station and leveled, theodolites can be used to measure vertical and horizontal angles. In modern theodolites vertical angles are measured from zenith line and is called zenith angle. The measurement and use of vertical angles will be treated somewhere else is this text (vertical surveys). The term “angle” between two intersecting lines (or vertical planes) as defined in introduction of this chapter, fig. 2.9 is referred or is used for horizontal angle, without specifying it to be horizontal. Horizontal angles are defined in clockwise direction from left to right.

Among the many methods of angle measurement (remember we mean horizontal angle) the selection of one will depend on 1) the purpose and accuracy requirement, 2) time and cost of measurement, 3) instrument and operator available, 4) topography, field and project conditions.

In the following paragraphs the four methods which will be used in this course will be illustrated. It is assumed that the theodolite is centered and leveled over the station A and pegs, poles or target plates are placed on targets.

3.1.4.1 Simple Method

To measure angle BAC:



i) sight to B with horizontal motion using optical sight, focusing and fine motions and read the horizontal circle and micrometer and record the reading say $b=10.310^g$
ii) turn and direct the telescope to C and read horizontal circle (if repeating theodolite use upper motion and its fine) and record $c=56.840^g$
iii) compute angle $BAC=c-b=56.840-10.310=46.530^g$

This method con be used with both repeating and direction theodolites.

3.1.4.2 Simple Method With Zero Reference

This method can only be used with repeating theodolites which provides two horizontal motions (UM&LM);

i) using micrometer knob set micrometer reading to exactly ($.000^g$) zero.
ii) loosen the upper motion clamp screw and rotate the instrument around vertical axis until you come (close) to zero grad on horizontal circle.
iii) tighten the upper motion clamp screw and use upper motion fine screw to bring horizontal circle reading to 0^g exactly. Now, horizontal circle and micrometer reading is 0.000^g exactly but telescope is sighting to an arbitrary direction.
iv) to bring telescope to target B (direction AB) use lower motion and its fine (this will not change reading on circle).
v) use upper motion and its fine to sight to C and read and record the circle reading c say c=46.532 which equal to angle BAC.BAC=46.532-0.000=46.532.

This method, though gives the angle (BAC) directly, requires more manipulation on the instrument and also remember the method can not be used with direction instrument. Besides in repeating theodolites if we confuse lower and upper motions and/or their fine motions, the whole measurement will be wrong.

3.1.4.3 Double Centering

The method measures angles in two positions of telescope namely direct and reverse. In some books these two positions are named as face left and face right. Measuring angles (taking horizontal circle readings) in two positions of telescope eliminates the error resulting from line of sight (LS) not being perpendicular to horizontal axis (HA). This error is called collimation (line of sight) error and can be eliminated automatically during the measurement with such an arrangement.

The procedure for double centering is as follows.

- i) sight to B and read the circle
- ii) sight to C and read the circle
- iii) tilt the telescope 180^o (200^g) around horizontal axis
- iv) rotate the telescope 180^o (200^g) around vertical axis
- v) sight to C and read the circle
- vi) sight to B and read the circle

The first two readings (on B and C) are taken during direct position of telescope and is called direct reading. In steps iii and iv we reversed the telescope and last two readings are reverse readings. Method can be used with both repeating and direction theodolites, but remember that if repeating theodolite is used, all horizontal motions are upper angular motions. The readings are shown in the following table:

Station	target	direct	reverse	mean	angle reduced
A	B	0.330	200.326	0.329	0.000
	C	46.862	246.854	46.858	46.529

Note that if a repeating theodolite is being used, direct reading on B can be set to zero, though it is neither necessary nor advised. As can be seen the difference between direct and reverse readings is about 200^g (180^o for degree instruments). The small differences in fractions of angles between direct and reverse readings might come from collimation error as well as from other operator errors listed at the end of this chapter. Computation of the angle is shown in the last two columns of the table.

3.1.4.4 Direction Method

This method of horizontal angle measurement can be applied using all types of theodolites. In repeating theodolites lower motion and its fine should not be used at all except to come to first direction with required approximate angle, at the beginning of each set.

This method makes it possible to eliminate (better to take the mean of) the division error of the horizontal circle as well as collimation and other random errors. In this method measurements are repeated to reduce the magnitude of random errors, but repeated measurements are carried out a different parts of horizontal circle around the entire arc in order to average the scale (division) error of horizontal circle as well as eccentricity of centers. The number of repetition which is called number of set is decided considering accuracy requirements of the work. Since half of the entire arc ($360^o = 400^g$) is covered by direct measurements and the other half by reverse measurements, it is sufficient to cover half of the circle by repeating measurement. Therefore, if n is number of measurement (# of set) then amount of shift angle (γ) between sets will be:

$$\Gamma = \frac{180^o\ (200^g)}{n}$$

The procedure is as follows: The theodolite is centered and leveled at A, directions AB, AC, AD and AE to be measured by 3 repetition, i.e. $\gamma = 200/3 \cong 70^g$

i) in direct position sight to B with horizontal circle reading around zero grad
ii) using upper motion sight to C and read horizontal circle
iii) using upper motion sight also to D and E and read circles
iv) reverse the telescope, sight to E and read circle
v) with upper motion sight to D, C and B and read circles
vi) (i)-(v) constitutes "one set", bring telescope to direct positions, set circle to around 70^g, come to A and repeat (i) to (v) and do the same for all other sets.
vii) Record and reduce (calculate) angles as shown below:

instrument station	station sighted	horizontal circle		mean	mean angle	Angle
		direct	reverse			
A	B	0.012	200.008	0.010	0.000	0.0000
	C	40.583	240.581	40.582	40.572	40.5733
	D	135.316	335.312	135.314	135.304	135.3050
	E	275.861	75.865	275.863	275.853	275.8493
A	B	70.006	270.010	70.008	0.000	
	C	110.579	310.583	110.581	40.573	
	D	205.310	5.308	205.309	135.301	
	E	345.858	145.856	345.857	275.849	
A	B	140.016	340.010	140.013	0.000	
	C	180.588	380.588	180.588	40.575	
	D	275.320	75.326	275.323	135.310	
	E	15.862	215.856	15.859	275.846	

3.2.5 Uses of Theodolite

Theodolites are employed principally for measuring horizontal and vertical angles. In some cases unknown angular values must be determined so that positions of points can be calculated. In others known angles are laid off to establish points at fixed locations given on construction plans.

Theodolite can be used along with liner (taping) measurements in many other field operations for measurement as well as to lay out construction plans e.g.,

1) to lay out grid for contouring and volume computation,
2) to establish lost or destroyed reference points,
3) to locate (find) the reference station in the field,
4) intersecting two straight lines,
5) locating points as described in fig. 2.10,
6) determining an inaccessible distance,
7) establishing a random line (when A and B can not be seen from any intermediate point in fig. 2.15),
8) measuring an angle when theodolite can not be set up at the station.
9) setting a monument (subsurface monument) to mark a survey point permanently.

3.2.6 Electronic Digital Theodolites

Modern technological advances have recently stimulated production of electronic digital theodolites that can automatically read and record horizontal and vertical angles. The recording of data can be done through electronic field databook or recently on memory cards both of which can be interfaced with computer for automatic computation and plotting.

These devices can be used exclusively for angle measurement, but often they are combined with an EDM and microcomputer to yield a so called TOTAL-STATION instrument. Total station units are also sometimes called electronic tachometers.

Coming back to electronic theodolites, except for their method of automatically resolving and recording angles, the mechanical operation of them is similar to that of standard instruments. To measure an angle, a backsight is taken using the clamp and tangent screws and an initial value entered into the display. Zero can be set (by simply pressing zero button) if direct angles are being measured but any required value may be entered if orienting on a line of known direction. The angle is then turned by pointing again, using the clamp and tangent screw and its value is automatically displayed in the instrument.

To eliminate instrumental errors and increase precision, angles can be repeated any number of times in both direct and reverse modes and the average taken. Built in computers will automatically perform the averaging and display results.

Some special capabilities designed into most electronic theodolites enhance their accuracy and expedite operation like,

1) built in computer that orients the vertical circle (for index error),
2) a device which senses any misleveling of the horizontal circle, whereupon the computer applies a correction to observed horizontal angles (before being displayed), thus the precise leveling of instrument is necessary,
3) the automatic system that averages values of diametrically opposed sides of the circles thereby correcting for any eccentric ties see 2.6.1.4 and also
4) a rough drive mechanism which permits advancing the circle to initiate readings for different positions and compensate for division error in horizontal circle.

Discussion and application of electronic theodolites, EDM and total stations will be given in chapters covering specific survey and specialized tasks.
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3.2.7 Sources of Errors in Angle Measurement

Like in any other survey, errors in theodolite works result from instrumental, natural and personal (operator) sources. It is impossible to determine the exact value of an angle and, therefore, and the (exact) error in its measured value.

Precise results (at required accuracy i.e., with allowable or tolerable errors) can be obtained, however, by

a) following specified procedures in the field
b) manipulating the instrument carefully and
c) checking measurements.

Most probable values and their mean errors can be computed from repeated measurements of angles.

3.2.7.1 Instrumental Errors

The reference lines or axes of theodolite were shown in fig. 2.12 which are referred to in the following discussion of instrumental errors.

1. Bubble axis is out of adjustment

If the axes of the plate bubble is not perpendicular to the vertical axes the VA will not be truly vertical when the bubble is leveled. This condition causes errors in measured horizontal and vertical angles that can not be eliminated by averaging direct and reverse readings. Bubble is out of adjustment if after centering and leveling it runs when the instrument is rotated 180o in the horizontal plane. The length for number of divisions of bubble run indicates double the tilt of the vertical axis, which is therefore made truly vertical by bringing the bubble back halfway using screws of the bubble tube.

2. Line of sight is not perpendicular to horizontal axis

If this error exists as the telescope tilted the line of sight generates a core whose axis coincides with the horizontal axis. The greatest error from this source occurs when tilting the telescope, as in extending a straight line or measuring deflection angles. This error is called collimation error and eliminated automatically by double centering and averaging equal numbers of direct and reverse readings.

The magnitude and sign of collimation error can be determined by taking half of the difference between direct and reverse readings to a sharp point.

e.g. Direct 35.286
Reverse 235.264
Collimation Error = $(35.286 - 35.264)/2 = 0.011^g$

Line of sight is inclined to the left in direct position of course the difference between direct and reverse measurements might result from errors other than collimation so measurements (D&R) should be repeated a few times to see magnitude of observation errors.

3. Horizontal axis not perpendicular to vertical axis

This axis error causes the telescope to rotate in an inclined plane as it turned around horizontal axis and accordingly will give fallacious horizontal angles when backsight and foresight have different angles of inclination. Errors from this source can be eliminated by direct and reverse readings.

4. Division errors and eccentricities of centers

This originates from the fact that the divisions of horizontal circle might have errors of position in varying magnitudes. The error in minimized (or averaged) by taking readings at several positions on the circle around the entire arc see 2.2.4.4.

5. Unsteady tripod

Tripod-leg bolts must be tight so there is neither play nor stain and the shoes set solidly in the ground. During storage, carrying and handling proper care must be given to protect tripod from rain, sun, dropping, inclined storage, etc.

6. Line of sight not parallel to attached bubble axis

Vertical circle should read 100 zenith angle when ABA is centered. Otherwise, an index error will be introduced in vertical angle measurement. Modern theodolites are not equipped with attached bubble axis.

7. Vertical cross hair is not vertical

This will produce error in horizontal angle measurements if any point, other than midpoint of vertical cross hair is used. The error can be detected by moving telescope u (or down) over a sharp point. If the point appears departed from cross hair, loosen two adjacent capstan screws and rotate the cross hair ring in the telescope tube and check again. We should note here that the instruments should be checked frequently for adjustment but their adjustment require expertise and can only be carried out by trained and qualified people.

3.2.7.2 Natural Errors

Weather, ground and other natural conditions will effect not only the measurement but the instrument and operators as well.

1. Wind. Wind vibrates the theodolite and deflects the plumb bob. On strong winds work should be suspended on precise work. Optical plummet is helpful in centering the instrument over the point on windy days.
2. Temperature effects. Variations in temperature cause unequal expansion of various parts of theodolite. Use of umbrella is advised on hot days especially if precise work is carried out.

3. Refraction. This is close to ground and cause bending of the light rays in vertical plane. It is desirable to keep lines of sight well above the ground and avoid sights close to buildings (horizontal refraction). For precise work make observations early in the morning or late in the afternoon.
4. Setting of the tripod. This will result for either loose ground or poor driving of shoes and will disturb both centering and leveling of instrument. In loose ground complete measurements as quickly as possible.

3.2.7.3 Personal Errors

We will make only a list of personal errors and remind you that whatever the operator does has an error in it.

- Instrument not set up exactly over a point.
- Level bubbles not centered perfectly.
- Improper use of clamps and fines.
- Poor focusing.
- Poor cross hair adjustment.

Poor focusing and improper cross hair adjustment will produce the paralox error which will effect pointing (sighting to target) which is an important source of error.

- Careless plumping of range pole or target
- Overly careful sights, checking and double checking the position of cross-hair setting on a target wastes time and produces poorer result than one fast observation. It means if you are overcareful you make more error.
- Target not sharp and clear.
- Reading errors.
- Micrometer setting error.

3.2.7.4 Mistakes

Mistakes result from the operator because he forgets confuses or neglects in operating the theodolite for measuring angles. Some common mistakes to guard against are:

1. Setting up over wrong point.
2. Sighting on wrong point.
3. Forget to level the instrument.
4. Forget to center instrument.
5. Forget to read or record the reading.
6. Forget to adjust cross hair.
7. Reading or recording an incorrect value.
8. Confusing horizontal & vertical arcs.
9. Forgetting to set micrometer.

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