

4.0 DIFFERENTIAL LEVELLING

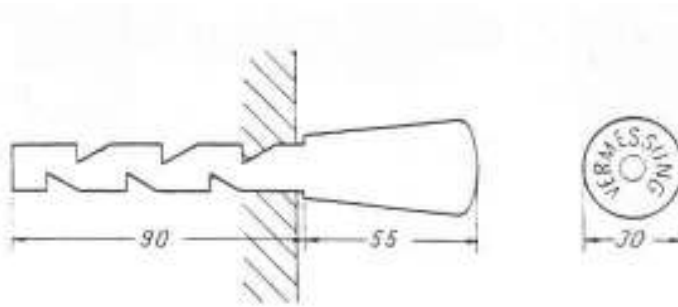


Fig. 4.0.1 Wall plug

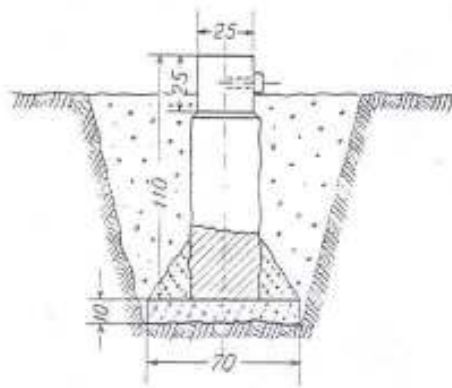


Fig. 4.0.2 Pillar plug

4.1 Bench Mark Levelling

4.1.1 General Rules

Bench mark leveling is used to densify an existing primary vertical network. The selection of instrument, rods, and procedure depends on the required accuracy.

To eliminate the influences of the earth curvature, refraction and residual instrumental errors, equal back-and foresights are to be used. To accelerate the work, two leveling rods should be utilized.

After solidly setting up the tripod, the instrument is secured on the tripod head. The tripod plate should be roughly level, so that the vertical axis is nearly plumb when the foot screws are turned equally. (For instruments with leveling tripods see section 2.4.3) The leveling is improved by using foot screws and bull's-eye bubble. Spirit levels then need be properly leveled, using foot screws or tilting screw as appropriate, while automatic levels do not require further leveling. The instrument remains on the tripod when moving to the next station. Spirit levels should be carried in such a manner that their vertical axis remains approximately vertical. Automatic levels, however, should be inclined to such an extent that the movable part of the compensator is arrested on one side rather than banging back and forth.

When placing the level into its container, all screws should be loosened, and slightly tightened after the instrument sits snugly. For longer transports, padded crates should be used.

The rod bubbles should be regularly checked and adjusted, especially after transport. It is important during transport to protect the graduated side of the rod.

4.1.2 Simple Leveling

In order to determine the elevation of a point B above the (curved) reference surface using bench mark A, the elevation difference $\Delta h_{A,B}$ is to be measured by simple leveling. For this, the distance AB (Fig.4.3) is subdivided by “turning points” into 70 to 100 m long sections, and the individual elevation differences h_1, h_2, \dots, h_n are observed and added together.

First the instrument station S_1 is selected halfway between A and TP_1 (the first turning point). Then the level is set up at S_1 and leveled. The rod man erects the rod at A, watching that the bull’s-eye bubble is centered. The observer then reads the backsight (B_1) after centering the spirit bubble or making sure that the compensator swings freely. The rod man proceeds now to TP_1 where he places a ground plate and then erects the rod. The observer now reads the foresight (F_1) after having checked and recentered the spirit bubble if required. The difference between back-and foresights provides the elevation difference, namely $h_1 = B_1 - F_1$, which completes the work at station S_1 .

Now, the rod at TP_1 is carefully rotated, and the instrument is moved to S_2 . Again, the backsight B_2 and – after moving the rod to TP_2 – the foresight F_2 are read. This provides $h_2 = B_2 - F_2$. This is continued. Therefore

$$\Delta h = h_1 + h_2 + \dots = \Sigma h = \Sigma \text{backsights} - \Sigma \text{foresights}.$$

If the elevation difference is positive, than the terrain rises, while it falls if Δh is negative.

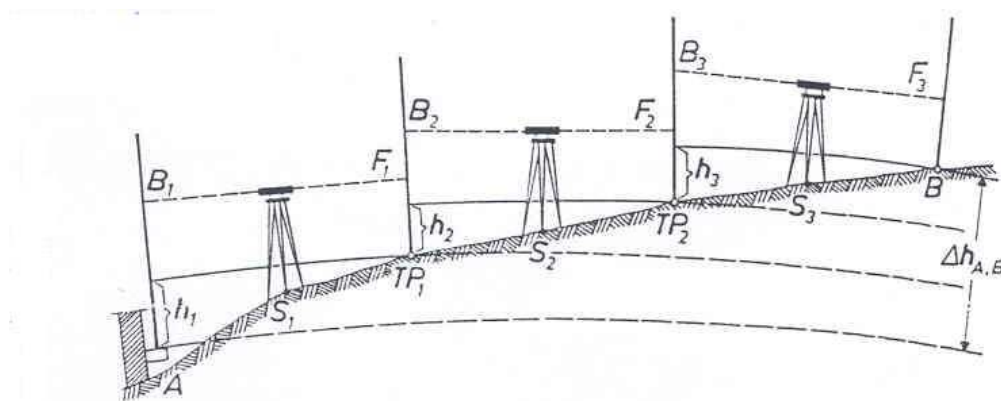


Fig. 4.1 Differential leveling

4.2 Differential leveling

In this method a horizontal line of sight is established by means of a level (instrument) vial or automatic compensator. A telescope (which can rotate in horizontal plane around the vertical axis) with suitable magnification is used to read vertical distances from fixed points on graduated level rods (staves). The basic procedure is illustrated in Fig. 4.2

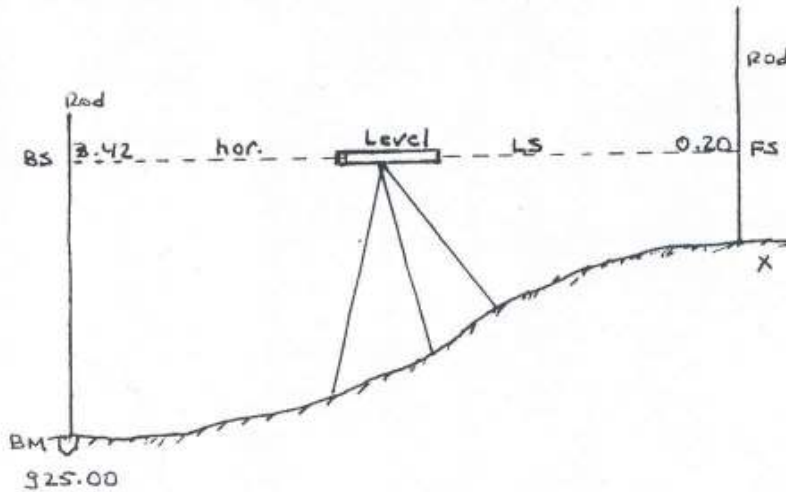


Fig.4.2 Direct differential leveling

$H_{BM} = 925.00$ m given elevation of BM on rock marked 3.42 m and 0.20 m rod readings on BM and X respectively.

$$\Delta H_{BM-X} = 3.42 - 0.20 = 3.22 \text{ m} \quad \text{elevation difference from BM to X}$$

$$H_X = H_{BM} + \Delta H_{BM-X} = 925.00 + 3.22 = 928.22 \text{ m} \quad \text{elevation of X}$$

$$H_X = H_{BM} + 3.42 - 0.20 = H_{BM} + BS - FS$$

$$H_X = H_{BM} + BS - FS$$

The maximum elevation difference which can be determined in this way will be about the length of rod (4.00 m usual). For larger elevation differences level is set up (a number of times) in between suitable selected minimum number of points and two rod readings are taken in each set up.

There are two commonly used techniques to record and reduce leveling notes. They are known as Rise and Fall and Height of Collimation. The observational procedure in the field is identical for either techniques, the only differences are in the lay out of the field notes and arithmetic method used to reduce and check the derived heights.

$$\text{Rise(+)} [\text{or Fall(-)}] = BS - FS$$

$$\text{Elev} = \text{Previous Elev} + \text{Rise (or Fall)} \quad \text{or similar to 3.5}$$

$$H_X = H_{BM} + (BS - FS)$$

While height of collimation method gives

$$HC = \text{Elev} + BS$$

$$\text{Elev} = HC - FS \quad \text{or similar to 3.5}$$

$$H_X = (H_{BM} + BS) - FS$$

4.2.1 Definitions

It is required to know the meaning of following terms in addition to those listed in 3.2 in order to understand the essentials and procedures of differential leveling.

Level : Name of instrument used in leveling which gives us a horizontal line of sight.

Focusing : Aiming the telescope on rod and then focusing it for a clear vision of rod.

Line of collimation : The line of sight defined by cross hair as appears in the objective of telescope.

Height of collimation : Elevation of line of sight obtained by adding backsight reading to the elevation of point on which BS reading is taken.

Backsight (BS) : The rod reading taken on the point of known elevation. It is the first rod reading at every level set up.

Foresight (FS) : The rod reading taken on the point of unknown elevation or last rod reading in each level set up.

Intersight (IS) : A (sight) reading between BS and FS on a point whose elevation is needed to be determined.

Turning Point (TP) : Also called Transfer Point (TP) or change point. Selected and used to transfer elevation between benchmarks. On every turning point one foresight and one backsight are taken. Turning points should be selected so that they are 1) identifiable 2) at equal distance from level. 3) at maximum distance 4) solid 5) sharp and 6) fixed. We are not actually interested to know elevations of turning points and accordingly they do not need to be definite and/or permanent points.

Point : By point in differential leveling it is meant to be the point on which the level rod (staff) is held. The point on which the level is set up is not of much concern in leveling except the level should be set up a equal distance (approximately) to BS and FS readings (points).

4.2.2 Level

The main requirement from a level is that it provides a horizontal line of sight. The line of sight is obtained through a telescope defined by cross hairs. The line of sight may be made horizontal either. 1) by means of a bubble tube placed parallel to the telescope axis (Line of sight) or 2) by means of an optical arrangements in the telescope (automatic compensators). Basically there are three differential types of level.

1. Dumpy (rigid) level : levels by the help of bubble tube.
2. Tilting level : small up/down motion of telescope is possible.
3. Automatic levels : levels automatically by compensators.

Basically and for most purposes; the levels listed above are totally adequate for leveling work. However, some manufacturers supply a horizontal circle for reading horizontal angles. Also on many modern levels, friction type clamping is used rather than providing a clamp screw.

4.2.3 Level Rod (Staff)

Leveling rods are graduated scale, held vertically over the points and viewed through the telescope of the level where the central horizontal hair of telescope cuts the rod is called the rod reading and this is equal to vertical distance between the line of sight of the level and the point on which the rod is held. Rods are usually made of wood although aluminum alloy and fiber glass rods are also available. The length of rod is generally in the range of 3-5 m and they may be hinged or telescopic for convenience in transporting.

The graduation is usually in alternate blocks and spaces of 10 mm with numbering in meters and decimeters. The subdivisions are painted in various colors on black on white, red on

white or red on yellow. The rods should be protected from sun, rain in inclined storage and drops and hits. The base plate should not be subjected to strong bangs or hits while putting the rod over a point.

4.2.4 Running A Line of Levels

In the preliminary example of direct leveling (Fig. 3.2), it was assumed that the difference of elevation between the two points considered could be obtained by a single setting of the level. This will be the case only when the difference in elevation is small and/or when the points are relatively close together. In Fig.3.3 the rods at the points A and K can not be seen from the same position of the level. If it is required to find elevation of point K from given elevation of point A, it will be necessary to divide the line AK into several sections and set up the level several times to establish turning points such as TP1, TP2, and TP3. A turning point is defined a solid, sharp and fixed point on which both a backsight and a foresight are taken on a line of levels. Small errors can be introduced depending on the kind of mark used as turning point while changing the face of the rod toward the level in each (different) set up. A rounded-top monument, steel piles, thin edge or a sharp stone make excellent turning points. In order to reduce the amount of work the minimum number of turning points should be selected. Horizontal distances to backsights and foresight should be made approximately equal by pacing, stadia measurement or by some other means. This will eliminate errors due to maladjustment of level (tilt error) and also the combined effects of the earth's curvature and atmospheric refraction.

Turning points need either to be described nor definite since they are merely a means to an end and usually will not have to be relocated. If possible, however, it is advisable to select some change points that can be relocated (identifiable), so if reruns are necessary because of blunders on long lines, field work can be reduced.

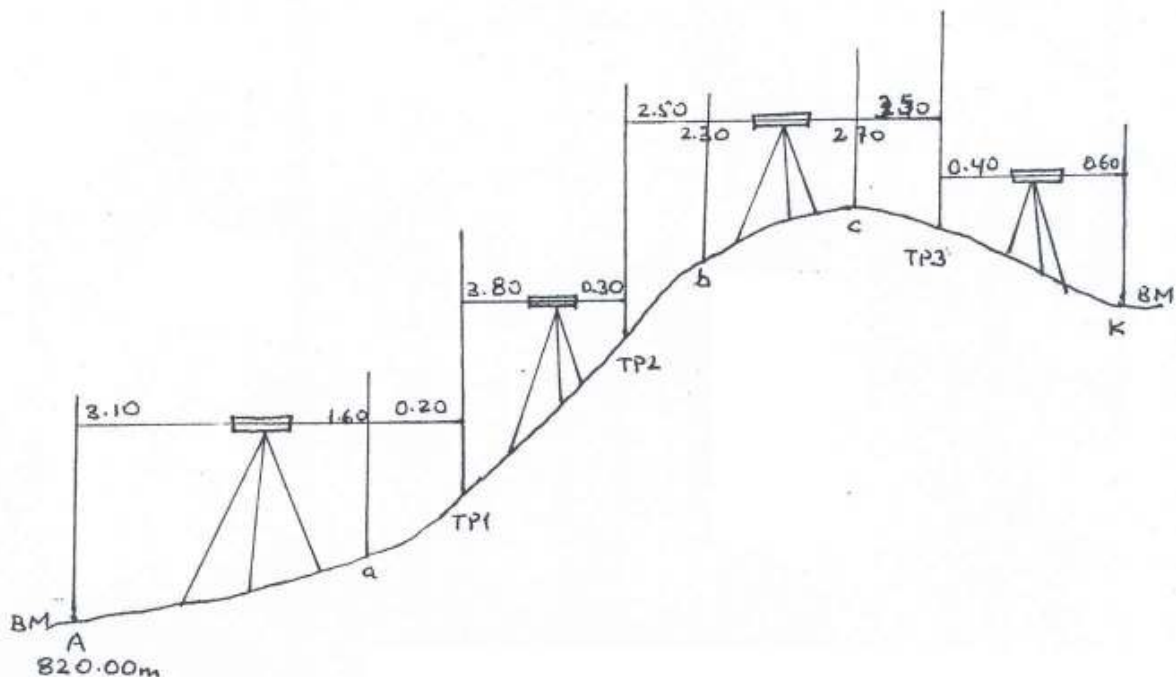


Fig. 4.3 Differential leveling

Fig. 4.3 sketches the procedure for differential leveling. In many cases it may be required and also economical to determine the elevations of some natural and/or artificial points of interest while carrying out a differential leveling operation. This can be achieved by holding the level rod over such points and take only one rod readings. These points are called intermediate points and intermediate sights or intermediate readings. Intermediate points, such as a, b and c as shown in Fig.4.3 are definite and of interest and need to be described in leveling notes. The

rod readings on intermediate points are called intermediate foresights (IS) and recorded in a separate column in notes as seen in table below. Sometimes the rod is held upside down for overhead points and rod readings are recorded as minus in corresponding column of leveling table.

4.2.5 Rise and Fall Method

As shown before (Fig. 4.3) the difference of backsight and foresight readings gives the elevation difference between successive points. The elevation difference from A to K in Fig. 4.3 then will be,

$$\begin{aligned}\Delta H_{AK} &= \sum(\text{BS} - \text{FS}) \\ \Delta H_{AK} &= \sum\text{BS} - \sum\text{FS} \\ H_K &= H_A + \Delta H_{AK} = H_A + \sum\text{BS} - \sum\text{FS}\end{aligned}$$

The elevation of each point is derived from the Previous point plus rise or fall between points. If the elevation of last point (BM) is computed, the following check can be carried out;

$$\sum(\text{BS} - \text{FS}) = \sum\text{BS} - \sum\text{FS} = \text{Last elev.} - \text{First elev.}$$

A similar check is also possible if the differential leveling is ended at the starting point which is called a loop leveling;

$$\begin{aligned}\sum\text{BS} - \sum\text{FS} &= \sum(\text{BS} - \text{FS}) = 0 \\ \sum\text{BS} &= \sum\text{FS}\end{aligned}$$

Example: reduce leveling notes

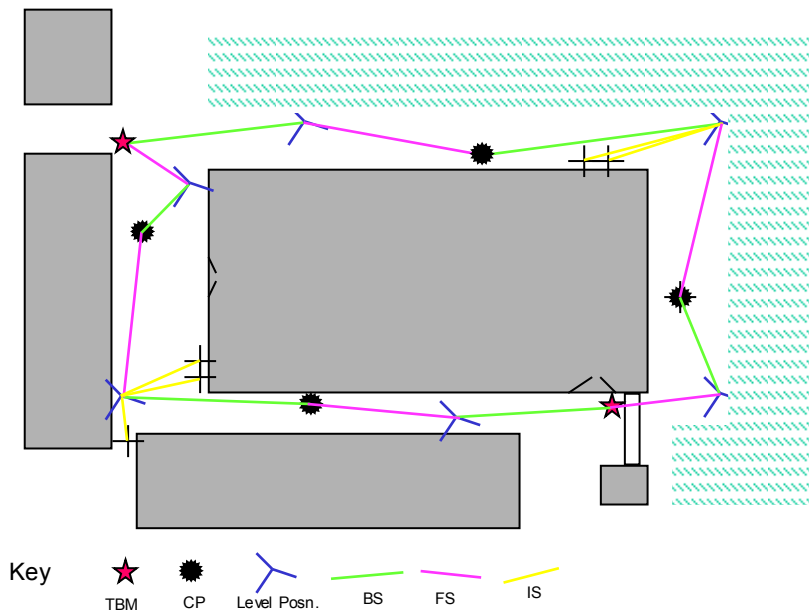
Point	BS	IS	FS	Rise (+)	Fall (-)	Elevation	Remarks
A	3.10					820.00	given
A		1.60		1.50		821.50	
TP1	3.80		0.20	1.40		822.90	
TP2	2.50		0.30	3.50		826.40	
B		-2.30		4.80		831.20	
C		2.70			-5.00	826.20	
TP3	0.40		3.50		0.80	825.40	
K			3.60		3.20	822.40	computed
Σ	9.80		7.60	6.60	4.40	-820.00	
						2.40	

$$\sum\text{BS} - \sum\text{FS} = 2.20 \text{ O.K.}$$

$$\sum(\text{BS} - \text{FS}) = 2.20 \text{ O.K.}$$

Table 3.1 Rise and fall method

4.2.6 Leveling Circuit



Site: Burnaby Building Instrument: L 52
 Date: 07/10/98 Observer: M.A.R.
 Weather: Good Booker: M.A.R.

BS	IS	FS	HPC	RL	Corr	Corr RL	Remarks
1.546			11.546	10.000			TBM 10.00m AAD
1.418		1.562	11.402	9.984			C.P.
	1.390			10.012			Point 1
	1.281			10.121			GL Struct. Lab Door
	-2.420			13.822			Top Struct. Lab Door
1.011		1.321	11.092	10.081			CP
		2.007		9.085			TBM 9.09m AAD

$$RL = HPC - FS$$

$$RL = 11.092 - 2.007 = 9.085$$

New staff
position
therefore
a new row

4.2.7. Height of Collimation Method

Height of collimation is defined as the vertical distance from the datum to the line of sight of instrument. H.C. is obtained by adding BS reading to the elevation of the point on which that BS reading is taken. The elevation of any point then is obtained by subtracting the rod reading (IS or FS) from H.C. for that level setup. The H.C. method is especially suitable where large

number of IS reading are taken during differential leveling. The following computation check is possible on H.C. method.

$$\sum BS - \sum FS = \text{Last Elev.} - \text{First Elev.}$$

Note that the checks of (3.7), (3.8), (3.9) and (3.10) are only done with BS and FS readings. Since no BS is taken on intermediate points (IS) any error or mistake on them will not be transferred to any other point.

Example: 3.4.2 (-) is for overhead reading: Reduce the notes.

Point	BS	IS	FS	H.C.	Elevation	
BM1	0.50			900.50	900.10	Given
A		1.40			899.10	
B		-3.20			903.70	
C		0.60			899.90	
TP1	0.80		2.10	899.20	898.10	
TP2	3.70		1.10	901.80	898.10	
D		-0.30			902.10	
BM2			0.60		901.20	
Σ	5.00		3.80		-900.00	
$\Sigma BS - \Sigma FS = 1.20$					$1.20 = \Sigma BS - \Sigma FS$	\checkmark checks

Table 3.2 Height of Collimation Method

4.3 Reciprocal Leveling

Topographic features such as rivers, lakes, gorges make it difficult or impossible to keep backsight and foresight short and equal. Thus unbalanced sights are unavoidable and accordingly tilt, curvature and refraction errors become significant. Reciprocal leveling is utilized of such locations.

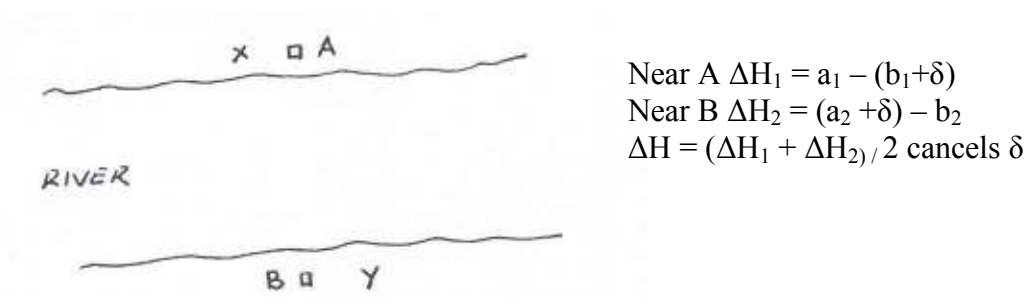


Fig.3.3 Reciprocal leveling

As shown in figure 3.3 a level is setup on one side of stream at X near to A and rod readings are taken on points A and B. Then the level is moved to Y near B and rod readings are taken on A and B again. Average of the two elevation differences is accepted as correct value. In eq. (3.11) δ combined effect of tilt, curvature and refraction.

4.4 Profile Leveling

On route surveys for highways and pipelines, for example, elevations may be required along the proposed centerline every 10, 20 or 3 m at critical points where slope or direction changes or at other main points such as bridges, culverts, curve points etc. When the elevations of the points and their distances are plotted on a Cartesian coordinate shows a profile – a line depicting ground elevations of a vertical section along a survey line.

Profile leveling like differential leveling requires establishing change points (critical points) on which both backsight and foresights are read. In addition several intermediate sights may be taken on points along the line from each instrument set up.

Since the variation of elevation is rarely as much as the increase in horizontal distance on the surface of the earth. Use of a larger scale for vertical distances is common. In practice 5x, 10x or up to 20x (20 times) larger scale can be used depending on topography of the area and type of project. See figure 3.4.

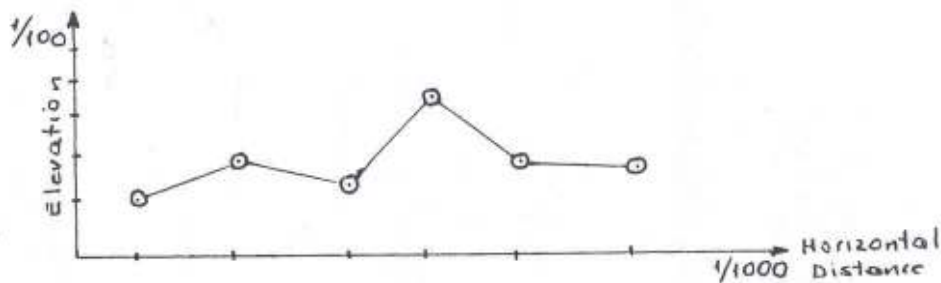
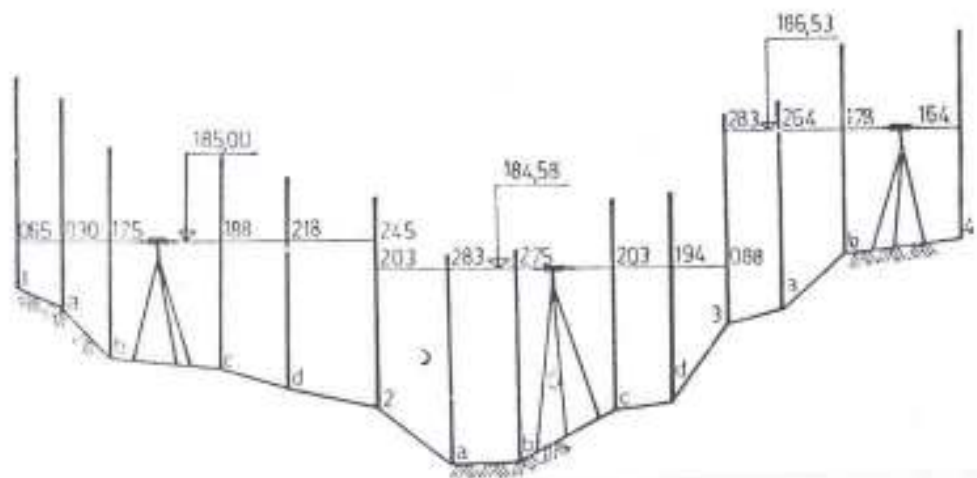


Figure 3.4 Profile along a route

Plotted profiles are used for many purposes such as (1) determining depth of the cut or fill on proposed highways, railways and airports (2) studying grade crossing problems and (3) investigating and selecting the most economic grade location and depth for sewers, pipelines, tunnels, irrigation ditches and other projects (4) balancing (making equal) cut and fill in a proposed route.



Profile leveling

Profile Leveling Records

P	Readings			Elevation	Height	Explanation
	BS	IS	FS			
1	0,65			184,35	185,00	
A		0,90		184,10		
B		1,75		183,25		
C		1,88		183,12		
D		2,18		182,82		
2	2,03		2,45	182,55	184,58	
A		2,83		181,75		
B		2,75		181,83		
C		2,03		182,55		
D		1,94		182,64		
3	2,83		0,88	183,70	186,53	
A		2,64		183,89		
B		1,78		184,75		
4			1,64	184,89		
.			.			
.			.			
.			.			

4.5 Checking and Adjusting Elevations

So far in tables (3.1) and (3.2) the arithmetic checks are shown on reduction of leveling observations. These checks do not show the mistakes and errors of operator. The only way in which the difference in elevation can be checked is either by carrying the line of levels from the last point (1) to a benchmark whose elevation is known or (2) to the original (starting) benchmark. These methods are named as (1) tied leveling and (2) loop leveling respectively. This procedure is called “closing the level circuit” and if there is a discrepancy between the calculated and known (given) elevation differences an error of closure exists and correction should be applied to the elevations of the points. The amount of correction to be applied to each point (skipping TP and IP) will be proportional either 1) to distances from the beginning of the level line or 2) to the number of instrument set ups between the beginning of the line and the point; if δ shows the closure error then

for loop leveling $\delta = \sum BS - \sum FS$

for tied leveling $\delta = (\sum BS - \sum FS) - (H_{last} - H_{first})$

Example 3.4.3 A line of levels in run from BM1 and BM2 whose elevations have been previously established as 180.52 m and 201.65m respectively. The following tabulation shows the measured elevations along this line of levels and the number set ups between points. Adjust the elevation of points.

Point	# of set up	Measured elevation (m)	Total # of set up	Correction	Corrected elevation (m)
BM1	5	180,52	0	-	180,52
A	12	189,26	5	-0,04	189,22
B	3	196,34	17	-0,13	196,71
BM2		201,80	20	-0,15	201,65
	17	-201,65			
closure error = $\delta = 0.15$					

$$\delta_1 = (15/20)*5 = 3,75 = 4 \text{ cm (-)}$$

$$\delta_2 = (15/20)*17 = 12,75 = 13 \text{ cm (-)}$$

$$\delta_3 = (15/20)*20 = 15 \text{ cm (-)}$$

example 3.4.4 The following tabulation shows a loop leveling where measured elevations of points and their distances from the starting point are given (distances between points might be given as well).

<u>Point</u>	<u>Distances from BM1</u>	<u>Measured elevations (m)</u>	<u>Correction (m)</u>	<u>Adjusted Elevation (m)</u>
BM1	0	80,96	-	80,96 fixed
A	150	87,65	+0,03	87,68
B	240	99,14	+0,05	99,19
BM2	400	80,87	+0,09	80,96 $\sqrt{\text{checks}}$
		-80,96		

closure error = $\delta = 0,09$

$$\delta_1 = (9/400)*150 = 3,38 = 3 \text{ cm (+)}$$

$$\delta_2 = (9/400)*240 = 5,4 = 5 \text{ cm (+)}$$

$$\delta_3 = (9/400)*400 = 9 = 9 \text{ cm (+)}$$

The magnitude of the maximum acceptable error of closure depends on the type and purpose of survey conducted. For ordinary leveling $\pm 20\sqrt{K}$ mm (where K is the length of level line in km) is commonly used. For example for problem in example 3.4.4 closure error must not exceed $\delta = 20\sqrt{0,4} = 4,74 \text{ mm} < 9 \text{ cm}$. and accordingly the leveling done would have to be repeated.

4.6 Sources of Errors in Leveling

Like all other measurements leveling measurements are subject to three sources of errors:

- (1) natural
- (2) personal and
- (3) instrumental

4.6.1 Natural Errors

- 1.1 Curvature and refraction errors: see action 3.3 for the effect of these errors on rod readings. Also remember that equalizing lengths of backsight and foresights cancel the errors due to curvature and refraction.
- 1.2 Temperature variations: Heat causes parts of levels to expand also liquid in bubbles expand and causes displacement. In hot days level should be protected by umbrella from direct exposure of sunlight. Also rod should not be left under sun when not used. Heat waves near the ground surface or adjacent to heated objects make the rod appear to wave and prevent accurate sighting. Raising the line of sight by high tripod set ups, taking shorter sights and avoiding any that pass close to heat sources and using lower magnification of a variable eye piece reduce the effect.
- 1.3 Wind: Strong wind causes the instrument to vibrate and make the rod unsteady and bent; stop leveling in windy days.
- 1.4 Settlement of the instruments: Settlement of the instrument after a back sight has been taken makes the foresight too small and therefore the recorded elevation of the next point too large. The error is cumulative in a series of setups on soft material. So care is required in setting up a level on spongy ground, some asphalt roads or ice. Reading must be taken in quick order, perhaps using two rods and two operators. Alternating the order of taking backsight and foresights helps somewhat.

4.6.2 Personal Errors

- 2.1 **Bubble not centered:** In working with tilting levels, errors caused by the bubble not being exactly centered at the time of sighting are most important, particularly on long sights. Bubble must be recentered before the foresight is taken. A good practice is to check the bubble before and after each sight (No need to do this if an automatic level is being used).
- 2.2 **Parallax:** Parallax caused by improper focusing of the objective and/or eyepiece lens results in incorrect rod reading. Parallax error can be checked by moving the eye in front of eyepiece and noticing whether the cross hairs are moving over the target.
- 2.3 **Faulty staff reading:** Incorrect rod reading result from parallax, poor weather conditions, long sights, improper rod location, interpretation of fraction and other causes.
- 2.4 **Rod handling:** Serious errors caused by improper rod handling (plumping) are eliminated by using a rod bubble that is n adjustment. Banging the rod on a TP between sights may change the elevation of point.
- 2.5 **Poor turning points:** If turning points are not selected and not used carefully serious errors are introduced. Soft and not sharp turning points will make the holding of the rod on the same point for both sights impossible.
- 2.6 **Mistakes:** a few common mistakes (of operator) are listed below
 1. Holding the rod at different places for BS and FS on a TP
 2. Reading a meter to a high
 3. Rocking the ordinary flat bottom rod while holding it on a flat surface (instead of a sharp point)
 4. Recording notes
 5. Touching tripod during reading process. If bubble is distributed after a BS center it and repeat BS reading. Do not touch (put your hand) while taking reading.
 6. To level bubble between BS and FS reading (see 5).
 7. Forget to level, forget to read or record.
 8. Sighting to wrong point
 9. Confuse BS and FS readings.
- 2.7 **Reducing errors and eliminating mistakes:** Errors in leveling are reduced (but never eliminated completely) by careful adjustment and manipulation of both instrument and rod and establishing standard field methods and procedures. The following routines most large errors or quickly disclose mistakes: (a) selecting and using TP's properly (b) checking the bubble before and after each reading (if an automatic level is not being used) (c) using a rod bubble (d) keeping the horizontal lengths of backsight and foresights equal (e) running lines forward and backward and (f) making the usual field-book arithmetic checks.

4.6.3 Instrumental Errors

- 3.1. Rod not correct length-similar to incorrect tape length.
- 3.2. Rod folding place error-effects reading above folding hinge.
- 3.3. Rod base plate distorted improper handling and banging of rod on TP's cause this error.
- 3.4. Tripod legs loose causes unstable setups.
- 3.5. Cross hair of level not exactly horizontal, reading at center of cross hairs will minimize this error.
- 3.6. Line of sight of level is not horizontal. This is called tilt error in the most important instrumental error in leveling. The tilt might be upward or downward and expressed in rod reading unit for a distance of 100 m. If the level is tilted upward it will give rod reading higher than the horizontal position and vice versa if tilted downward. The tilt error can be determined by so called "two peg test"

and rod readings can be corrected for tilt errors. As mentioned before errors due to tilt are canceled if the horizontal lengths of backsights and foresights are kept equal.

4.6.3.1 Two Peg Test

The first task that should be undertaken by an operator is to see if the level is adjusted correctly. i.e., is the line of sight horizontal when the bubble is centered. This check known as two peg test, should be made every time a leveling instrument is taken from the store room or if the same instrument is being used for an extended period it should be checked on a routine basis e.g. at the start and end of a run of levels. The test is simple to perform and should take an experienced operator no longer than 10 minutes.

As shown in fig. 3.5 the level is set up at half way between two pegs A and B which are say 60 m apart. Readings are taken on a rod held at each peg in turn and since the instrument is equidistant from each, the observed difference in height $3.616 - 2.794 = 0.822$ m will be the true height difference since the effect of tilt of line of sight will be the same on both rod readings.

The instrument is now being moved as close to one of the pegs as possible (min focusing distances). The difference in elevation is again observed.

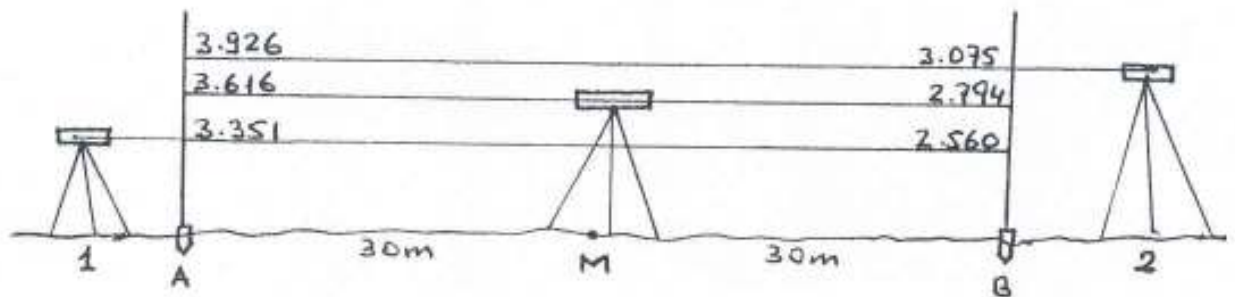


Figure 3.5 Two Peg Test

The level is now being moved close to B and same thing is repeated (this extra measurements for eliminating mistakes and averaging other errors)

Position of Level	Rod Reading on		Computed ΔH
	A	B	
M	3,616	2,794	0,822
1	<u>3,351</u>	<u>2,560</u>	$0,791 - 0,822 \rightarrow 31\text{mm}$
2	<u>3,926</u>	3,075	$0,851 - 0,822 \rightarrow 29\text{ mm}$ mean = 30 mm

$$\text{tilt error for 100 m} = (30/60) * 100 = 50 \text{ mm}$$

When the level is at 1 the error of rod reading on A (3,351) is zero and also when the level is at 2 the error of rod reading on B (3,075) is zero since the level is very close to the rod for these cases). The underlined rod readings in the table have the tilt errors which causes 31 mm and 29 mm errors in elevation differences. To determine the direction of tilt (up or down) we ask the question whether we need to increase the values (2.560 and 3.926) or to decrease them in order to make the elevation differences for second and third setups equal to that of first set e.g

$$3.351 - \underline{2.560} = 0.791$$

In order to increase 0,791 to 0,822 m (correct) the reading 2,560 must be decreased this means telescope is tilted upward similarly $3,926 - 3,075 - 0,851$ in order to decrease 0,851 to 0,822 the reading 3,926 must be decreased and therefore the tilt is upward

tilt error $\delta = 30$ mm upward/100 m

As an example the rod reading s in the table of the two peg test can be corrected as follows:

$3,616 - (0,050/100)*30 = 3,616 - 0,015 = 3,601$	
$2,794 - (0,050/100)*30 = 2,794 - 0,015 = 2,779$	$\Delta H=0,822$ m
$3,351 - 0 = 3,351$	$\Delta H=0,821$ m
$2,560 - (0,050/100)*60 = 2,560 - 0,030 = 2,530$	
$3,926 - (0,050/100)*60 = 3,926 - 0,030 = 3,896$	$\Delta H=0,821$ m
$3,075 - 0 = 3,075$	

It is important to note here that the difference between 31 mm on 29 mm is because of other observational errors.

Summary:

Place two pegs about $L= 30$ m (40m) apart

Set up level midway between the two pegs.

Read staff on each peg, and calculate true height difference.

Collimation Error e =difference in the difference and is expressed as a number of mm per L m.

Acceptable errors: 1 mm per 20 mm.

Test should be carried out regularly say once per week or two.