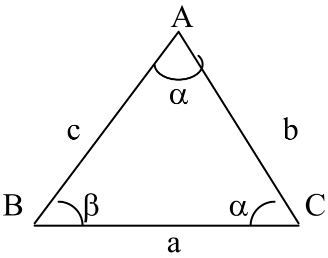
CHAPTER 5

5.0 AREA COMPUTATION

1) Triangle



1) $s = \frac{a+b+c}{2}$

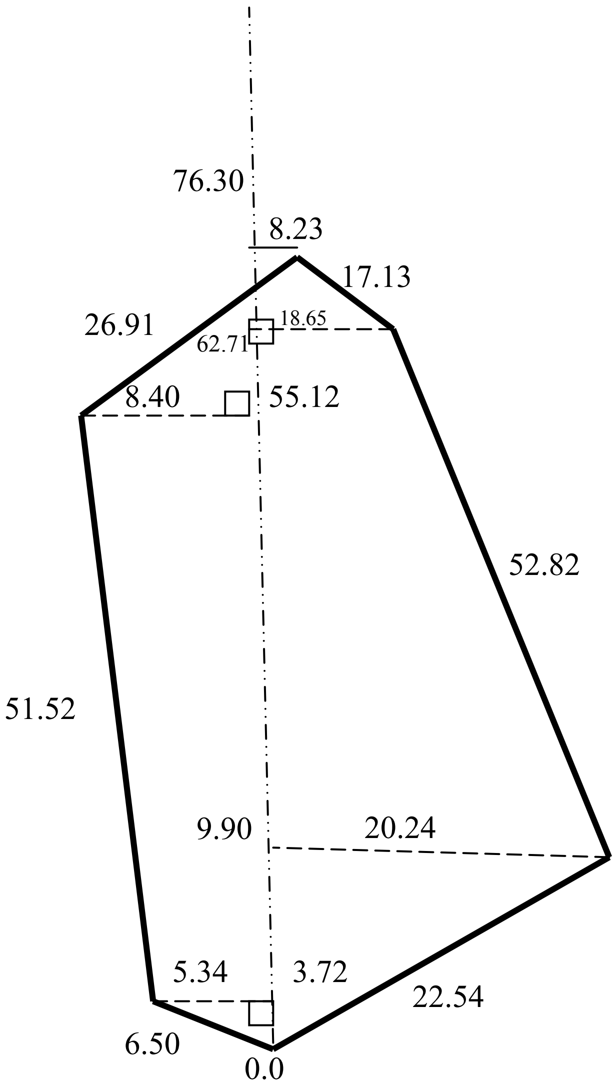
$$F = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

2) $F = \frac{1}{2}\, a.b \sin\delta$

3) $F = \frac{1}{2}\, a.h$

2) Trapeziums

$$F = \frac{1}{2}\,(a + b)*h$$



ORTHOGONAL SURVEY

5.1 Area Determination

If a closed figure, bounded by straight lines, is surveyed according to the procedures given in the previous sections, its area can be computed. The method applied depends on the type of survey as well as on the required accuracy. The following basic procedures are available:

1) Numerical area determination
2) Semigraphical area determination
3) Graphical area determination

5.1.1 Numerical Area Determination

5.1.1.1 Area determination Using Field Values

This approach is suitable if the area was surveyed in such a manner that it can be easily divided into triangles and trapeziums. The area of these figures is equal to half the base line times the height. As shown in Fig. 5.1.1, a reversed trapezium follows the same rule, as the area is equal to the difference of the two triangles:

$$A = \frac{1}{2}\,a(h_1 - h_2)$$

Where a is the base line, h_1 is the height inside the area, while h_2 is the height outsight the area.

Rather than carrying the factor ½ usually double the area is computed and divided by 2 after completion of the computation. Often the accuracy requirements are such that the factors can be rounded to decimeters.

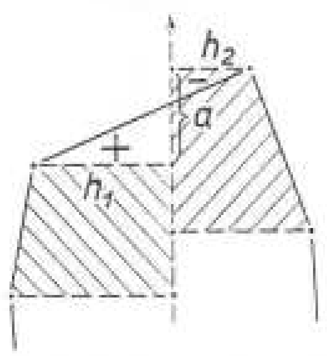


Fig. 5.1.1

Example: Area determination for the lot shown in figure page-1

a) with centimeter accuracy					b) rounded to decimeter		
(3,72 – 0,00).(5,34 + 0,00)	=	3,72. 5,34	=	19,86	3,7.5,3	=	19,6
(55,12 – 3,72).(8,40 + 5,34)	=	51,40.13,74	=	706,24	51,4.13,7	=	704,2
(76,30 – 55,12).(8,40 – 8,23)	=	21,18. 0,17	=	3,60	21,2. 0,2	=	4,2
(76,30 – 62,71).(8,23 + 18,65)	=	13,59.26,88	=	365,30	13,6.26,9	=	365,8
(62,71 – 9,90).(18,65 + 20,24)	=	52,81.38,89	=	2053,78	52,8.38,9	=	2053,9
(9,90 – 0,00).(20,24 + 0,00)	=	9,90.20,24	=	200,38	9,9.20,2	=	200,0
				3349,16:2			3347,7:2
			A=	1674,58 m^2		A=	1673,8 m^2

5.1.1.2 Area Determination Using Coordinates

This approach is easily explained using the practical situation given in fig….. , however, the origin is mentally shifted to the left such that it falls outside the figure (fig. 5.1.2). Then the area is obtained by subtracting the area 1'-1-6-5-4-4' from 1'-1-2-3-4-4':

$$2A = (x_1-x_2)(y_1+y_2)+(x_2-x_3)(y_2+y_3)+(x_3-x_4)(y_3+y_4)+(x_4-x_5)(y_4+y_5)$$
$$+(x_5-x_6)(y_5+y_6)+(x_6-x_1)(y_6-y_1)$$

or: $2F = \Sigma(x_i-x_{i+1})(y_i+y_{i+1})$

Field Surveys with Simple Instrumentation and their Evaluation

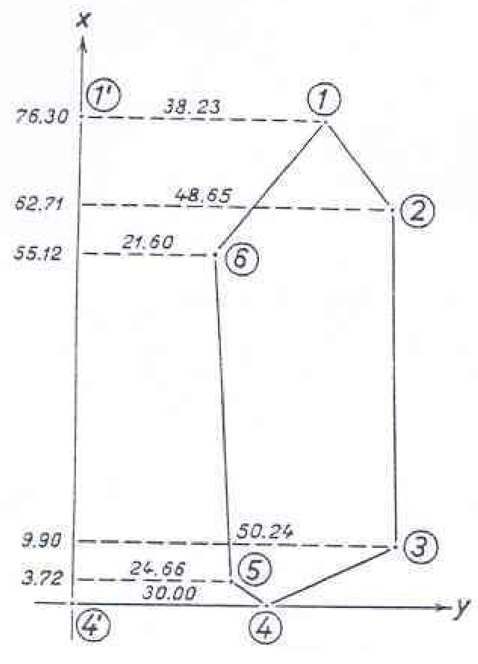


Fig.5.1.2 Gauss's area formula

Similarly, by exchanging the x- and y- axes:

$$2A = \Sigma(y_{i+1}- y_i)(x_i+x_{i+1})$$

These formulae are called the “Gauss trapezium formulae”. After multiplication and rearrangement the “Gauss triangle formulae” are obtained:

$$2A = \Sigma x_i (y_{i+1} - y_{i-1}) = \Sigma y_i (x_{i-1} - x_{i+1})$$

Example: Compute the area of the lot shown in fig.5.1.2 using the first triangle formula:

Point	y_i	x_i	$y_{i+1}-y_{i-1}$	$x_i \Delta y_i$
1	38.23			
2	48.65	62.71	12.01	753.15
3	50.24	9.90	-18.65	-184.64
4	30.00	0.00	-25.58	
5	24.66	3.72	-8.40	-31.25
6	21.60	55.12	13.57	747.98
1	38.23	76.30	27.05	2063.92
2	48.65			3349.16:2
			A=	1674.58 m^2

5.1.2 Semigraphical Area Determination

The semigraphical area determination (Fig. 5.1.3) is based on the following consideration: Measuring errors *da* and *dh* are present in a triangle with a short base a and a long height h. The area error is then $dA = ½(adh + hda)$. In order to keep dA small, the error da of the short base a is to be kept small as it is multiplied by the large value h.

A semigraphical approach is suitable if the area is divided into narrow triangles with short base lines, measured accurately in the field. The long heights can then be extracted from an scaled plan with lower accuracy.

Example: The area of the lot shown in Fig. 5.1.3 is

2A = 19,95.42,0 + 17,48.40,2 + 19,52.81,6 + 17,98.66,6 + 18,91.33,0 = 4955; A=2478 m^2.

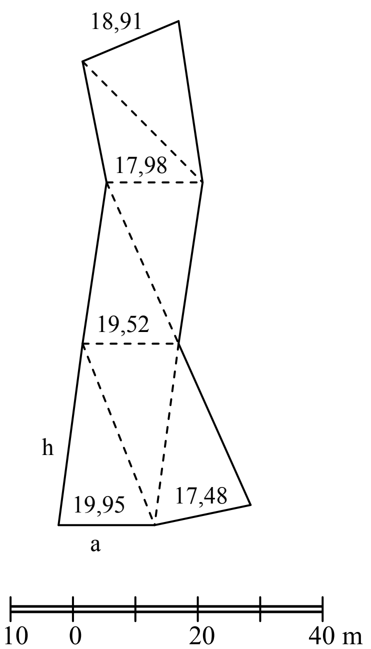


Fig.5.1.3 Semigraphical area determination

5.1.3 Graphical Area Determination

5.1.3.1 Graphical Area Determination Using Simple Tools

Graphical methods are used when numerical methods are not feasible (e.g., for natural boundaries), as well as for independently checking both the computation and accuracy of the plotting of the plan.

Regular figures are divided into triangles or trapeziums whose base length and heights are measured in the plan with a ruler.

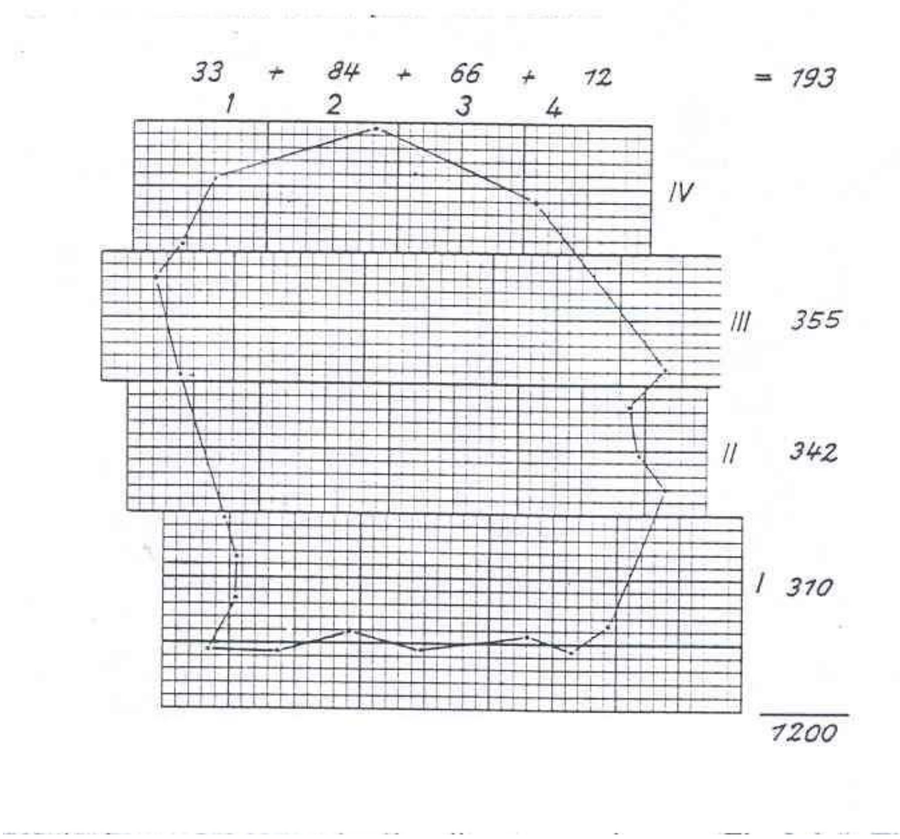


Fig. 5.1.4 Use of graph paper for area determination

Irregular figures are covered with millimeter graph paper (Fig. 5.1.4). The area is then obtained by adding the number of squares and multiplication with the scale factor.

Example:

		mm^2
Strip I (left to right)	=	310
Strip II (right to left)	=	342
Strip III (left to right)	=	355
IV/1 + IV/2 + IV/3 + IV/4 = 33 + 84 + 66 + 10	=	193
	Total	1200

For long narrow figures, a parallel line graph (Fig. 5.1.5) is more suitable. A transparent sheet with parallel separated by h is placed normal to the length of the figure dividing it into a number of small trapeziums. With m_1, m_2, …. m_n being the median lines of these trapeziums, the area becomes

$$A = hm_1+hm_2+.....hm_n = h\Sigma m.$$

The individual medians are mechanically added with a compass. h equals 10 in the reduced Fig. 5.1.6 Σm amounted to 247.5 leading to A=2475 m^2 as compared to 2478 m^2 obtained with the semigraphical approach. It should be noted that irregular end figures have to be adjusted as shown in Fig. 5.1.5.

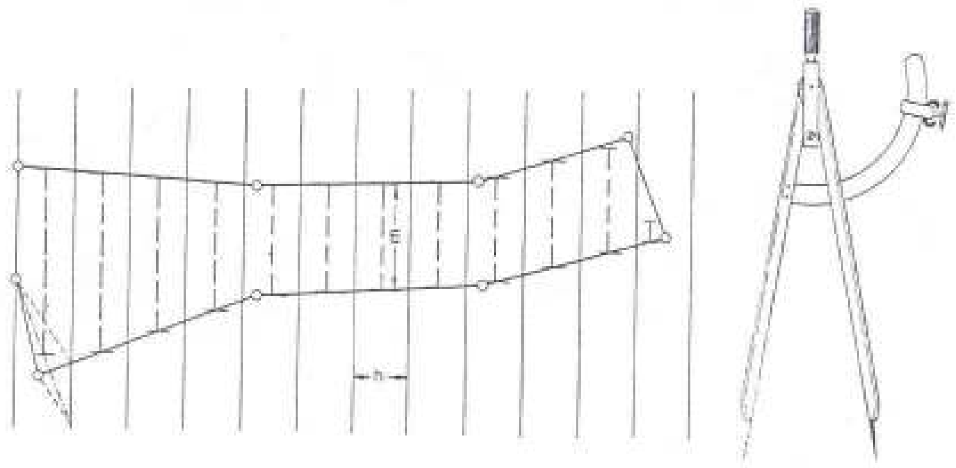


Fig. 5.1.5 parallel line graph

5.1.4 Graphical Area Determination Using The Planimeter

The polar planimeter a mechanical integration instrument which permits the determination of an area by following its boundary lines.

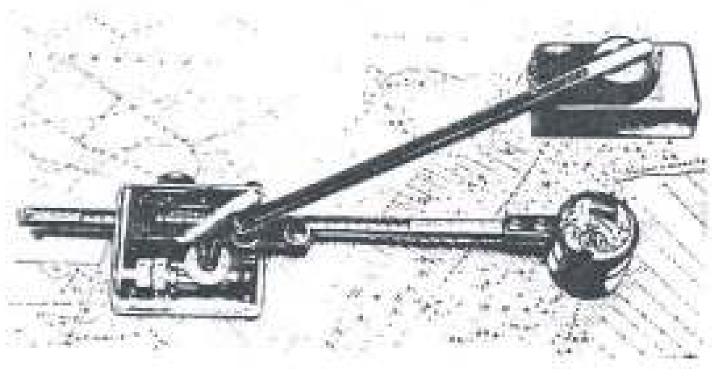


Fig. 5.1.6

5.2 CROSS SECTIONS – PROFILES – EARTHWORK COMPUTATIONS

5.2.1 CROSS SECTIONS

Cross sections are straight vertical sections through the earth's surface normal to the center line. They are staked with simple tools as indicated in section 8.2. At deflection points the angle is bisected, while in curves it is normal to the tangent. The lateral extension of cross sections is between 20 and 50 m, depending on the purpose and type of terrain. Cross sections are needed for the geometric design work and also provide the data for earthwork computation. Therefore, they have to be measured wherever there are significant changes in the terrain and where the direction of the center line changes. The lateral distances are measured in decimeters from the center line and recorded in a hand sketch (not to scale, Fig. 5.2.7). The elevations are determined with a construction level which is usually set up in such a manner that the whole cross section can be measured from one station. The height of the instrument is determined by reading the rod at the center line point. The rod readings are entered into the cross section sketch. If, as in fig. 5.2.7, the whole profile cannot be measured from one station, then the instrument has to be set up again, and another reading at the center line point is required. It is possible to obtain measuring and computational checks by an independent second survey with a slightly different height of instrument. However, the direct comparison between terrain and rod readings make large blunders unlikely, which is why a second survey is not usually carried out. Occasionally, cross sections are measured with a horizontal rod (Fig. 5.1.3), if the slope is extremely steep or in dense forest. Cross sections in rivers are tied to the shore. A measuring string or line with meter sections is used for distances. For narrow rivers, this line is stretched horizontally and serves as the height reference. The water surface is used as the reference for wider rivers and the depth is measured with poles. The water surface itself is determined by tying the top of posts near the shore into a bench mark and then measuring down along this pole.

Remarks to the field notes: Starting at the origin A, the instrument height for each station is obtained by adding the backsight reading to the MSL elevation of the back point. By subtracting foresights or ground readings from this instrument height, the elevation of turning points or intermediate points is obtained. Thus for perfect leveling, the given MSL height of the end point E should be obtained. A closure error within the required accuracy is afterwards distributed to each station. The elevations of intermediate points are not controlled by this page check, they can, however, be checked by using the rise-fall approach as mentioned in section 5.2.2.2. Only a second independent survey will check the actual measurements.

Table 5.2.1

Profile record

Pt.	*d*	Backsight	ground reading	Foresight	H.I.	MSL elev.	corr. (mm)
1	1	34	4	5	6	7	8
A	50	1,415				49,675	
	50			1,290	51,090		
TP_1	45	0,420				49,800	3
	45			1,655	50,220		
0^{+0}	50	1,390				48,565	6

0^{+25}		3,175			49,955	46,780	6
0^{+50}		2,955				47,000	6
0^{+75}		0,595				49,360	6
1^{-0}	50			0,700			
	50	2,040				49,255	9
1^{+25}		1,545			51,295	49,750	9
1^{+50}		3,235				48,060	9
1^{+75}		0,820				50,475	9
	50			0,415			
2^{+0}	30	1,005				50,880	12
	30			2,090	51,885		
TP_2	40	1,840				49,795	15
	40			1,235	51,665		
E						50,430	18
		8,140		7,385		40,448	
			0,755			0,773	

5.2.2 Plotting of Profiles and Cross Sections

Based on field notes (Table 5.2.1) and field sketch (Fig.5.2.7), profiles and cross sections are plotted on graph paper. The profile shown in (Fig. 5.2.8) is based on a reference line which is parallel to the horizon and a multiple of 10 meters above the reference surface. The center line is usually plotted in the same scale as the planimetric plans with a 10 times or higher scale increase for elevations. Thus the vertical characteristics are displayed prominently. The x-coordinates are the station number along the center line. Common scales for planimetry are 1:1000 to 1:5000, with 1:100 to 1:250 for elevations.

The designed structure is also entered (usually in a different color), so that elevation differences between the local terrain and the planned structure can be obtained from the profile.

For cross sections (Fig.5.2.9), a uniform scale for planimetry and elevations is chosen, usually the same as the height scale in the profile. The reason for this is the direct determination of cross sectional areas as well as having true angular relationships which help in the design.

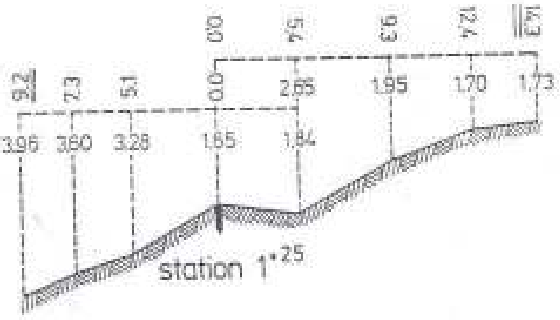


Fig. 5.2.7 Cross section sketch

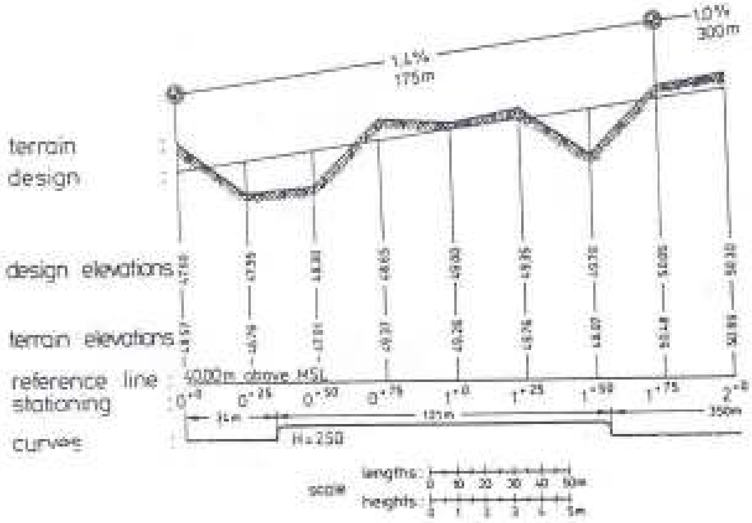


Fig. 5.2.8 Profile plot

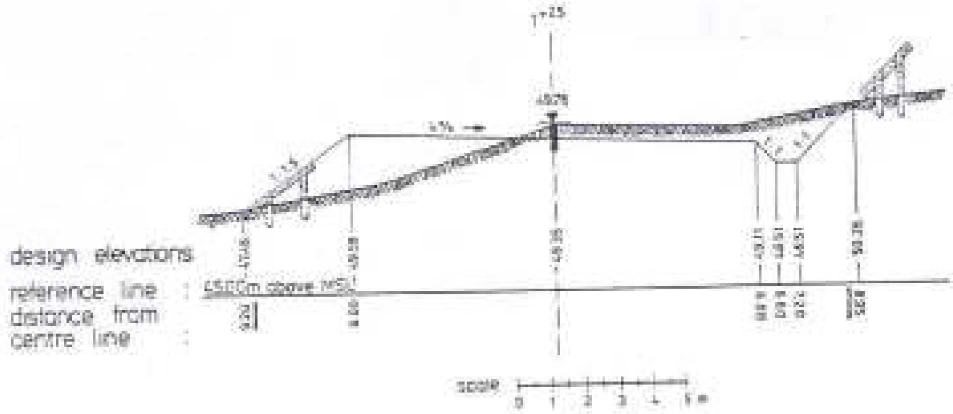


Fig. 5.2.9 Cross section plot

The center line stake is transferred from the profile, then the instrument height is lightly drawn, and all readings are plotted as negative offsets. For space reasons, it is common to raise the reference line more than for the profiles. By superimposing the design cross section (use of templates) the cross sectional are representing cut and/or fill is defined and can be determined.

The earthwork is usually computed as the product of the average cross sectional area between two subsequent cross sections, times their separation (section 14.2).

5.2.3 Area Leveling

(see also section 14.1.3.2)

For certain types of planning and construction (e.g. sports fields, drainage systems, etc.) profiles and cross sections provide insufficient coverage. Instead, contour lines are required in the plan. They are plotted based on points which are locally coordinated in both positions and height. In flat terrain leveling is used, while stadia methods (4.2) are preferred in more varied terrain.

5.2.3.1 Positioning

Positioning depends on the purpose of the plans and the terrain characteristics. Usually, available plans or maps are sufficient.

a) If a site plan with numerous points is available (boundary markers, etc.), some of these points are selected for leveling, and necessary additional ones for heights are referenced from the existing points.

5.2.4 Differential Levelling

b) If only the boundaries of a large lot are known, then profiles are measured across the lot whose end points are tied into the boundaries.(fig.5.2.10)

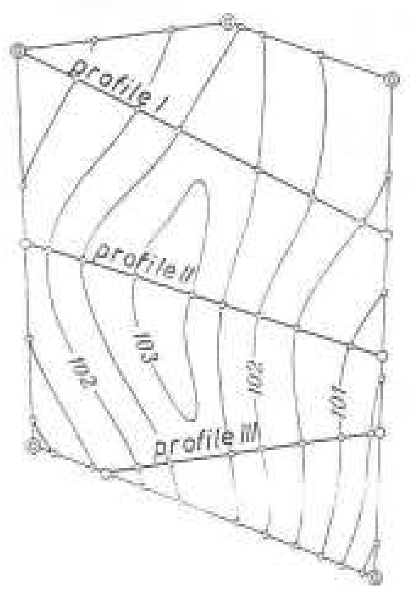


fig 5.2.10 Tie-in profiles

c) If there is no plan available, the area is covered with a rectangular grid, usually with equal separation (fig.5.2.11) of 10 or 50 metres. This is especially useful if earthwork volumes have to be computed. Significant terrain points can be referenced within this grid frame. The points are usually staked and numbered.
d) In a somewhat more varied terrain, the important points are determined by stadia leveling (section 9.1.2.5.), which utilizes the same kind of point selection as regular stadia surveys.

5.2.4.2 Elevation Measurements

Elevation measurements consist of level lines tied into bench marks, using characteristic terrain or profile points as either turning points or ground readings. Simple leveling with a construction level is sufficient for this purpose. In method d)(section 9.2.4.1) stadia distance and angles are measured as well.

Due to the lower accuracy requirements, sighting distance of up to 300m can be used.

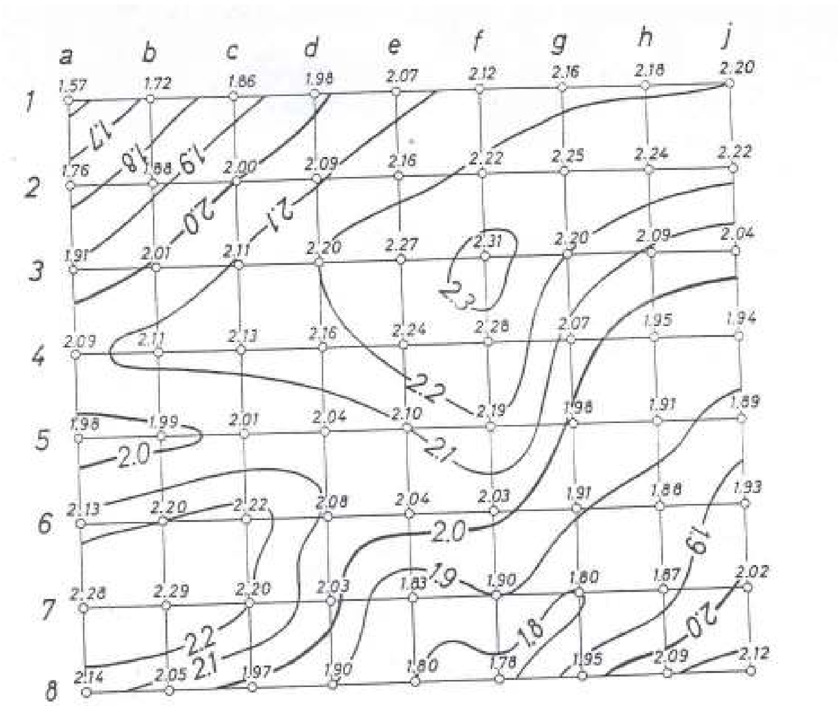
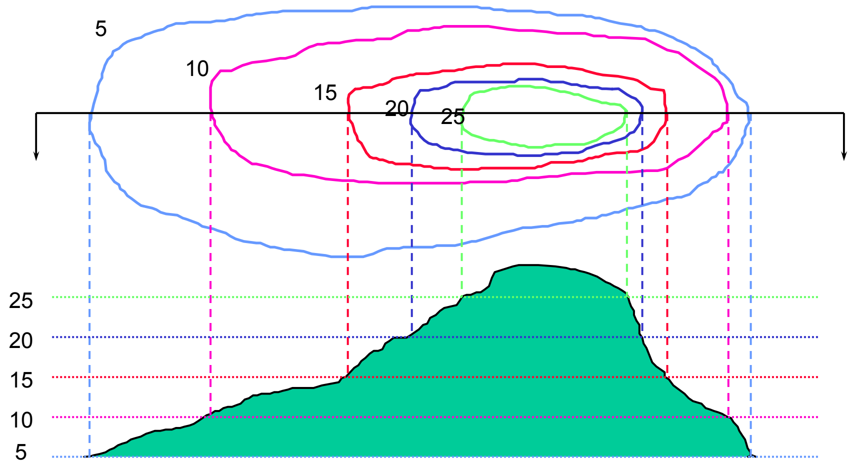


fig. 5.2.11 Leveling grid

5.2.4.3 Plotting of Contour Plans

First the positions of all plotted onto the planimetric plan. The elevations are not beside each point in preparation for the generation of contour lines. Contour lines are horizontal curves which connect points of equal elevation.

A contoured spoil heap.



Depending on the required accuracy, they are plotted at 0.1,0.5,or1m intervals in flat areas, and 2.5or5m in moderately undulating terrain in such a manner that points of these elevations are interpolated from the measured terrain in such a manner that points and then connected by a smooth curves(fig. 5.2.11).Interpolation is performed along the line of steepest slope by either numerical or graphical means. In fig. 5.2.12 two terrain points (1) and(2) with h_1 =63.3 and h_2 =68.5 are located at a distance of 28.3mm from each other on the plan. It is desired to obtain the intersections of line (1),(2)with the contours 64.0,65.0,…,68.0, thereby assuming that the straight line connection between (1) and (2) represents the terrain(linear interpolation).

If in Fig. 5.2.12, the distance from (1) to the contour 64.0 is denoted as S_4, to the contour 65.0 as S_5, etc., then the following values are obtained by simple slide rule or calculator operation:

$$S_4 = \frac{28.3}{5.2}\, 0.7; \qquad S_5 = \frac{28.3}{5.2}\, 1.7; \qquad S_6 = \frac{28.3}{5.2}\, 2.7;etc.$$

or

$$S_4 = 3.8\ ; \quad S_5 = 9.2\ ; \quad S_6 = 14.7\ ; \quad S_7 = 20.1; \quad S_8 = 25.6 \text{ mm.}$$

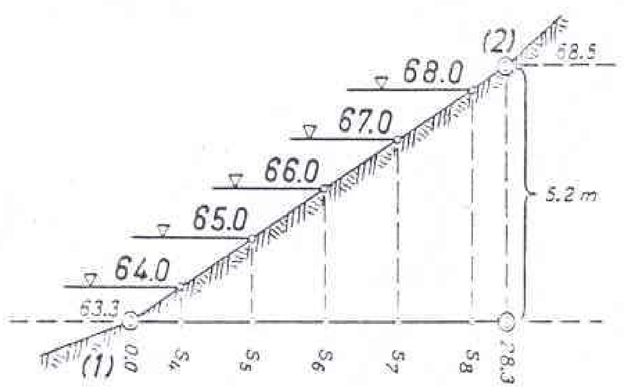


Fig. 5.2.12 Interpolation of contours

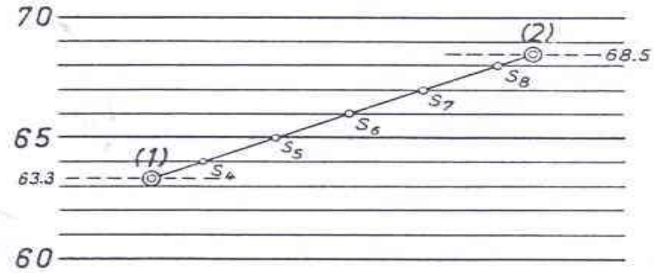


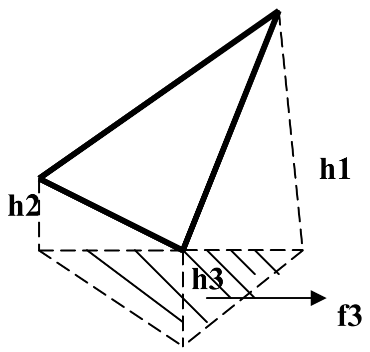
Fig. 5.2.13 Graphical interpolation

For the graphical interpolation (fig.9.2.13) a sheet of transparent millimeter graph paper is placed on top of the plan in such a manner that point (1) obtains a value corresponding to 3.3, and point (2) to 8.5. then a ruler is placed along points (1) and (2), and the points of intersection between rule edge and respective graph lines is pricked onto the plan and subsequently numbered.

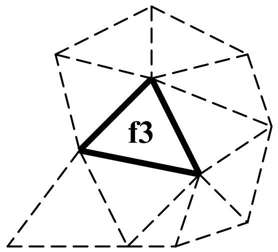
Occasionally doubts arise as to the best course of a contour line. It is therefore recommended to prepare a field sketch during the survey, indicating the approximate course of contours as well as the direction of maximum slope to aid the interpolation. Contours with even elevations are drawn as thicker lines to provide a better overview and to serve as index contours.

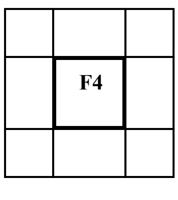
EARTHWORK COMPUTATION USING AREA LEVELING

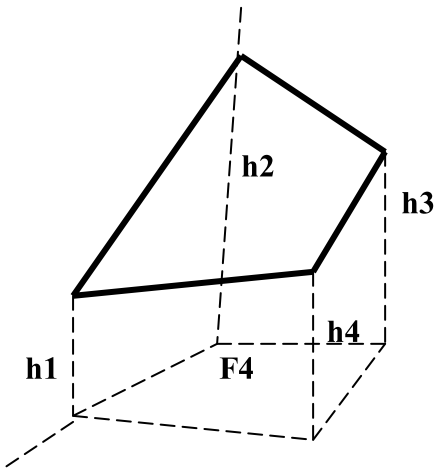
In moderately rolling terrain , an area is designated for a sports field ,factory yard, parking lot, etc..., and thus has to be transformed to a horizontal plane. For such a project, usually a square grid is superimposed onto the terrain as the base for area leveling. If such an arrangement is not suitable, a continuous series of triangles can be used instead. All corner points are leveled or else measured by stadia.



$$V_3 = F_3 \frac{h_1 + h_2 + h_3}{3}$$







$$V_4 = F_4 \frac{h_1 + h_2 + h_3 + h_4}{4}$$

Triangular grids generally provide a better approximation to the terrain than square grids. For road planning, it is possible to have one triangle side coincide with a cross section.

Position and elevation of the grid points are best determined by tacheometric means, preferably with a recording tacheometer. Thus the directions(α), distance(s), and zenith angles (s) from the instruments station P_0 to the vertices of the triangle F_3 are to be observed. The elevations of the vertices are obtained as

$$h_i = h_{Po} + s_i \cot z_i + a - i$$

Where h_{Po} is the height of the instrument station, a the instrument height above terrain, and i the target height or the rod reading. The area F_S is computed by either

$$2F_3 = S_1.S_2.\text{Sin}\ (\alpha_2-\alpha_1)+S_2.S_3(\alpha_3-\alpha_2)+S_1.S_3(\alpha_1-\alpha_3)$$

The relative error of the volume can be kept within

$$\frac{\Delta V}{V} = 0.5\%$$