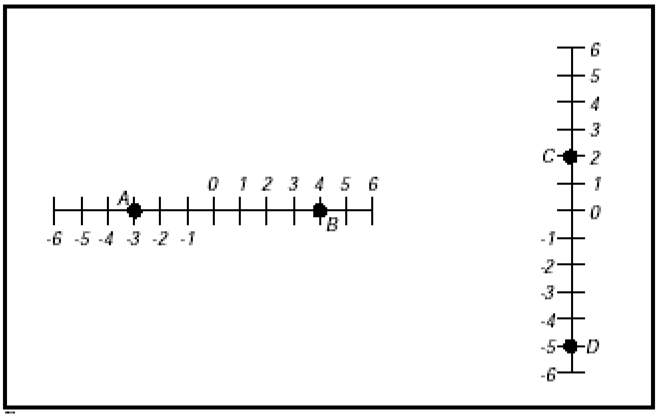
COORDINATES AND COORDINATE COMPUTATIONS

In Surveying, one of the primary functions is to describe or establish the positions of points on the surface of the earth. One of the many ways to accomplish this is by using coordinates to provide an address for the point. Modern surveying techniques rely heavily on 3 dimensional coordinates.

In order to understand the somewhat complex coordinate systems used in surveying, we must first look at the Rectangular Coordinate System (or Cartesian Plane) from basic mathematics.

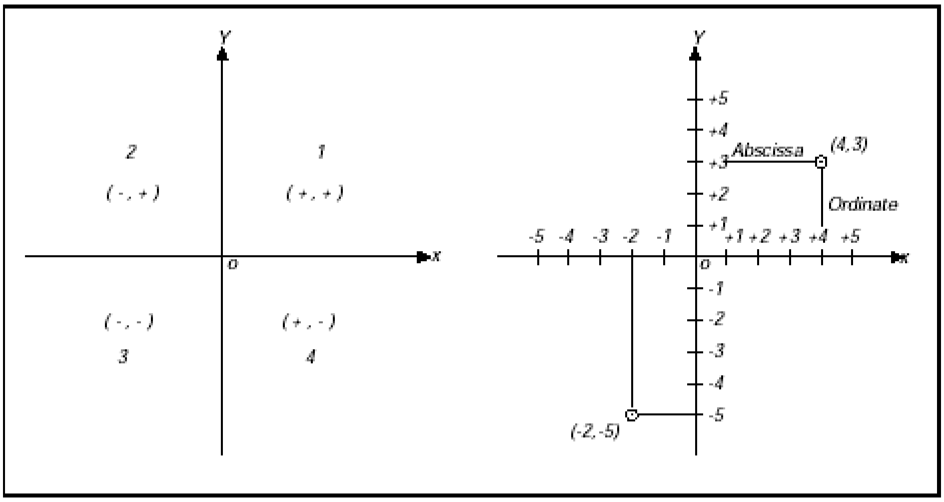
To keep it simple let's start by looking at a 1 dimensional system for locating points. Consider the horizontal line shown on the left of figure 32. A point on the line marked "0" is established as the origin. The line is graduated and numbered (positive to the right of the origin and negative to the left). Any number can be plotted on this line by its value and distances to other points on the line can be easily calculated. If all of our work was done precisely along a line, this system would be sufficient.

We live in a 3 dimensional world therefore we need a better system. Let's look at a 2 dimensional system for locating points. The right of figure 32 shows a similar graduated line but in a vertical position. This line would function in a similar way as the horizontal line but giving locations of points in a different direction. By coinciding those lines at their respective origins we provide the foundation for a rectangular coordinate system.

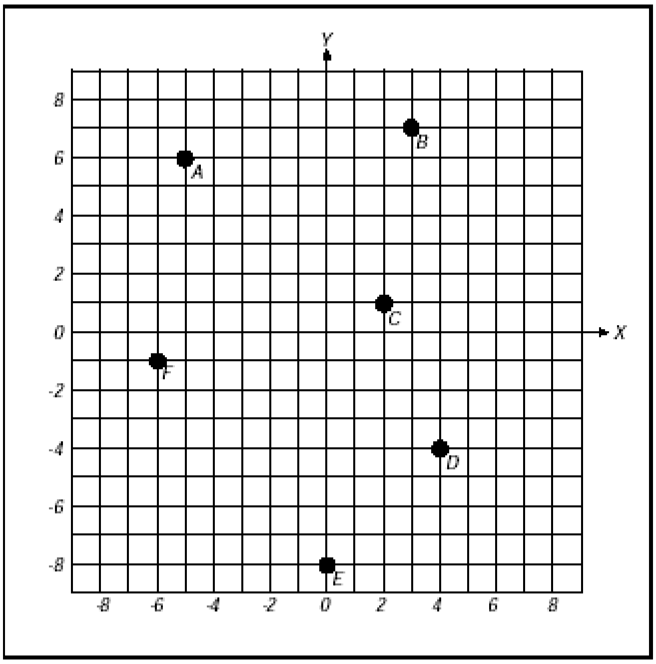


In the right of figure 33, is what is described as a *rectangular coordinate system*. A vertical directed line (y-axis) crosses the horizontal directed line (x-axis) at the origin point. This system uses an ordered pair of coordinates to locate a point. The coordinates are always expressed as (x,y). The horizontal distance from the y-axis to a point is known as the *abscissa.* The vertical distance from the x-axis is known as the *ordinate.* The abscissa and ordinate are always measured from the axis to the point - never from the point to the axis. The x and y axes divide the plane into four parts, numbered in a counterclockwise direction as shown in the left of figure 33. Signs of the coordinates of points in each quadrant are also shown in this figure.

Note: In surveying, the quadrants are numbered clockwise starting with the upper right quadrant and the normal way of denoting coordinates (in the United States) is the opposite (y,x) or more appropriately *North, East.*



Determine the coordinates of the points shown in the figure below.



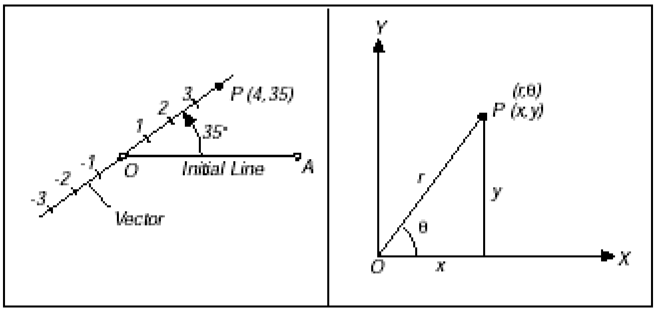
Point	X	Y	Point	N	E
A			D		
B			E		
C			F		

Polar Coordinates

Another way of describing the position of point *P* is by its distance *r* from a fixed point *O* and the angle θ that makes with a fixed indefinite line *oa* (the initial line). The ordered pair of numbers (r,θ) are called the polar coordinates of *P*. *r* is the radius vector of *P* and θ its vectorial angle. Note:

(r, θ), (r, θ + 360o), ($-r, \theta$+ 180o) [the grade system is used in surveying, 400gone=360o] represent the same point.

Transformation of Polar and Rectangular coordinates:
1. $x = r\cos\theta y = r\sin\theta$ (if θ and r are known)
2. r= sq($x2 + y2$) θ= tan-1 (y/x) (if x and y are known)



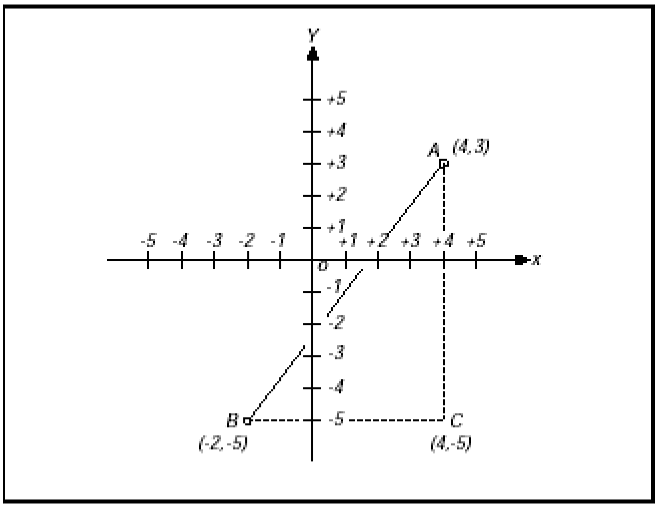
Measuring distance between coordinates

When determining the distance between any two points in a rectangular coordinate system, the pythagorean theorem may be used (see Review of Basic Trigonometry). In the figure below, the distance between A and B can be computed in the following way :

AB = [4-(-2)]*2*+[3-(-5)]*2* AB = [4+2)]*2*+[3+5)]*2* AB = 10

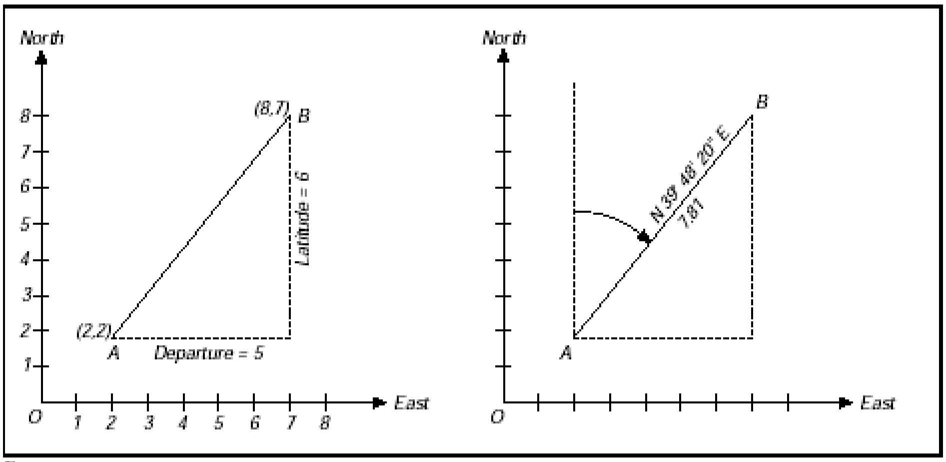
CB=+4-(-2)=4+2 AC=+3-(-5)=3+5

Point C in this figure was derived by passing a horizontal line through point B and a vertical line through point A thus forming an intersect at point C, and also forming a right triangle with line AB being the hypotenuse. The x-coordinate of C will be the same as the x-coordinate of A (4) and the y-coordinate of C will be the same y- coordinate of B (-5).



Inverse

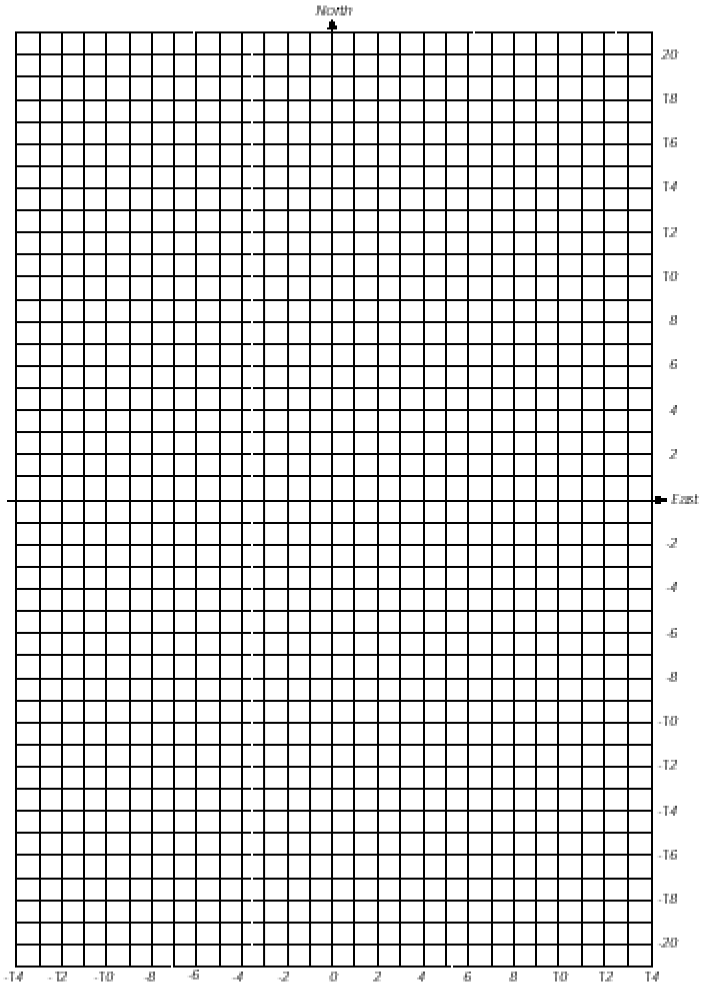
In mathematics, the coordinates of a point are expressed as (*x,y*). In surveying, as mentioned earlier, the normal way of denoting coordinates (in the United States) is the opposite (*y,x*) or more appropriately *North, East.* The difference in Eastings between 2 points is referred to as the *departure* and the difference Northings is the *Latitude.* To *inverse* between points means to calculate the bearing and distance between 2 points from their coordinate values. Start by algebraically subtracting the Northings to get the Latitude, and the Eastings to get the Departure. A simple right triangle is formed and the pythagorean theorem can be used to solve for the hypotenuse (distance between points). To find the bearing we need to calculate the angle from the North/South line at one of the points by using basic trigonometry.



Plot the following points (N,E) and connect with lines in the following order ABCDEA.

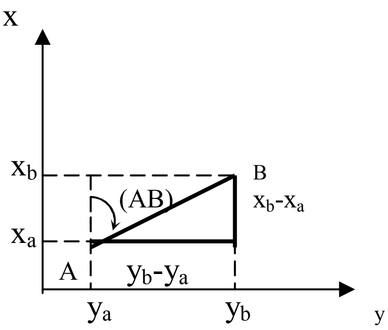
A (12,6) B (-14,12) C (-12,1) D (-3,-9) E (16,-10)

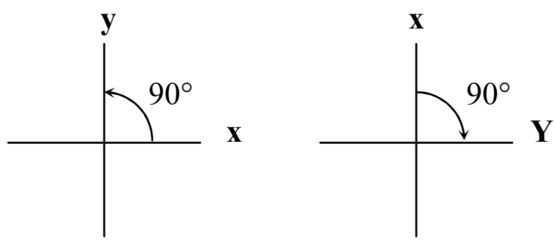
2. Find the bearing of each line (i.e. AB, BC, etc.) and the perimeter distance.



GEODETIC PROBLEMS

GEODETIC PROBLEM I





" IN MAHTS"   "IN SURVEY"

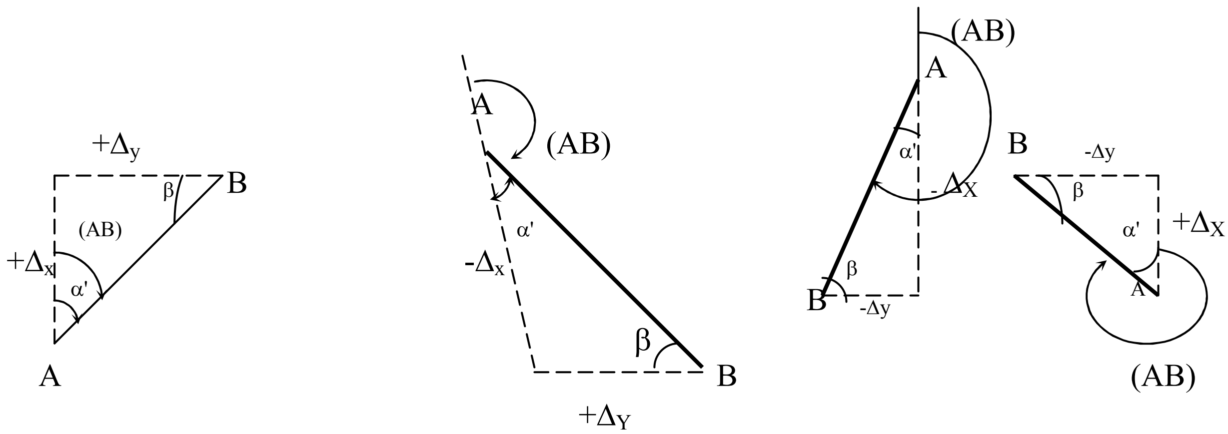
$A(x_A, y_A)$: American
$A(y_A, x_A)$: German, as Turkish

- GIVEN $A(x_A, y_A)$, $\underbrace{s = AB,\ \alpha_{AB}}_{\text{WANTED}}$

- COMPUTE $B(y_B, x_B)$

$$\text{Tan}(AB) = \frac{y_b - y_a}{x_b - x_a} = \frac{\Delta_y}{\Delta_x}$$

; α_{AB} = arc tan (AY / Δ_x)
α' = arc tan (|AY| / $|\Delta_x|$)
α' = (Bearing angle)



	$+\Delta_Y$	$+\Delta_x$	(AB)	(AB)
1.	+	+	α'	$100 - \beta$
2.	+	-	$200-\alpha'$	$100 + \beta$
3.	-	-	$200+\alpha'$	$300 - \beta$
4.	-	+	$400-\alpha'$	$300 + \beta$

1. $$\frac{\Delta_Y}{\Delta_X} \rightarrow \frac{+}{+} \rightarrow (AB)=\alpha'$$

2. $$\frac{\Delta_Y}{\Delta_X} \rightarrow \frac{+}{-} \rightarrow (AB)=200-\alpha'$$

3. $$\frac{\Delta_Y}{\Delta_X} \rightarrow \frac{-}{-} \rightarrow (AB)=200+\alpha'$$

4. $$\frac{\Delta_Y}{\Delta_X} \rightarrow \frac{-}{+} \rightarrow (AB)=400-\alpha'$$

$$\overline{AB} = \frac{y_b - y_a}{\sin (AB)} = \frac{x_b - x_a}{\cos (AB)}$$

or;

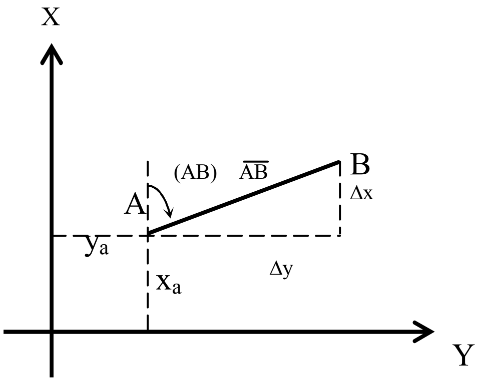
$$\overline{AB} = \sqrt{\Delta y^2 + \Delta x^2} = \sqrt{(Y_B - Y_A)^2 + (X_B - X_A)^2}$$

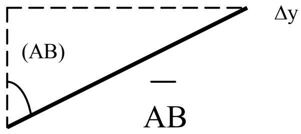
GEODETIC PROBLEM II

GIVEN $A(X_A, Y_A)$ WANTED $B(X,Y)$

$\overline{AB}$

(AB)





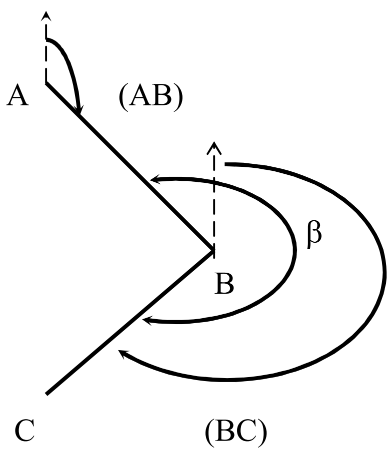
$$\sin(AB) = \frac{\Delta y}{\overline{AB}}$$

* $\Delta y = Y_b - Y_a = \overline{AB}.\mathrm{Sin}(AB)$

* $\Delta x = X_b - X_a = \overline{AB}.\mathrm{Cos}(AB)$

* $Y_b = Y_a + \Delta_y$
  $X_b = X_a + \Delta_x$

1. STANDARD PROBLEM III



$(BC) = (AB) + \beta \pm 200$

* if $(BA) + \beta > 200^g$ 1

$(BC) = (AB) + \beta - 200$

*if $(AB) + \beta < 200^g$ 2

$(BC) = (AB) + \beta + 200$

* If the computed value is bigger than 400^g in the second case, 400^g must be Subtracted from the computed value.

<u>Ex.1</u>

(AB) = 171,4075	(BC) = (AB) +β ± 200
β = 244,3618	(BC) = 415,7693-200
	(BC) = 215,7693

<u>Ex.2</u>

(AB) =71,4821	(BC)=175,2240-200
β =103,7419	(BC)=375,2240

<u>Ex.3</u>

(AB)=336,9175

β =346,4139

(BC) = 683,3314-200

(BC) = 483,3314

(BC) = 83,3314

2. STANDARD PROBLEM IV

GIVEN A (X_A,Y_A)

B (X_B,Y_B)

C (X_C,Y_C)

WANTED (AB),(BC),(AC)